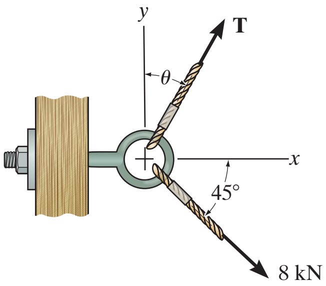
•2–1. If $\theta = 30^\circ$ and $T = 6$ kN, determine the magnitude of the resultant force acting on the eyebolt and its direction measured clockwise from the positive x axis.



The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively. Applying the law of cosines to Fig. *b*,

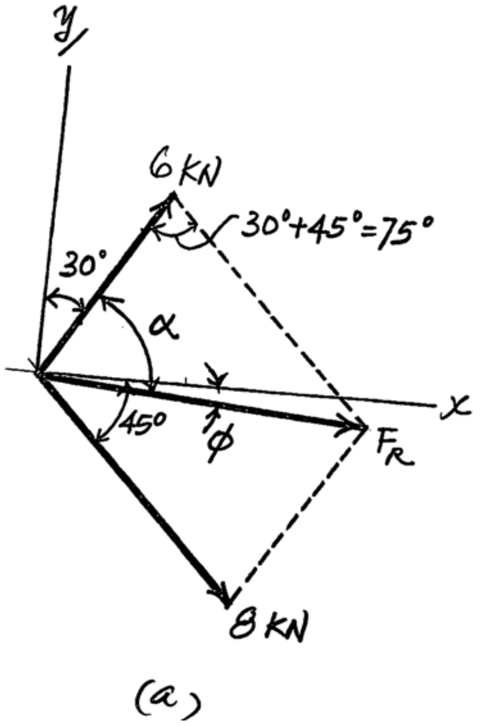
$$F_R = \sqrt{6^2 + 8^2 - 2(6)(8)\cos 75^\circ}$$
$$= 8.669 \text{ kN} = 8.67 \text{ kN} \qquad \textbf{Ans.}$$

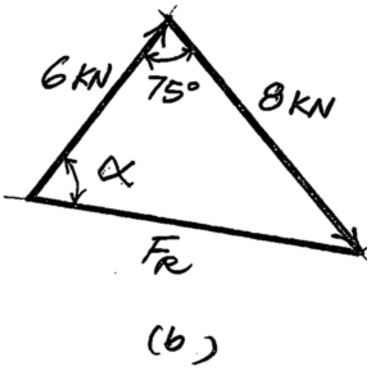
Applying the law of sines to Fig. *b* and using this result , yields

$$\frac{\sin\alpha}{8} = \frac{\sin 75^\circ}{8.669} \qquad \alpha = 63.05^\circ$$

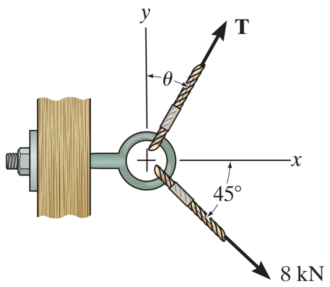
Thus, the direction angle ϕ of $\mathbf{F}_R$ measured clockwise from the positive x axis is

$$\phi = \alpha - 60^\circ = 63.05^\circ - 60^\circ = 3.05^\circ \qquad \textbf{Ans.}$$





2–2. If $\theta = 60°$ and $T = 5$ kN, determine the magnitude of the resultant force acting on the eyebolt and its direction measured clockwise from the positive x axis.



The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of cosines to Fig. *b*,

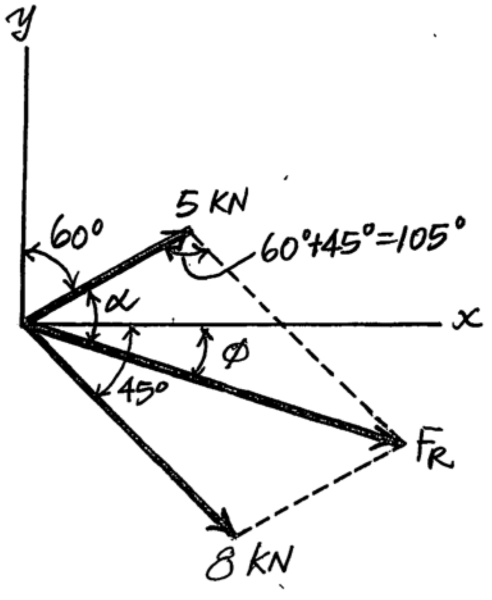
$$F_R = \sqrt{5^2 + 8^2 - 2(5)(8)\cos 105°}$$
$$= 10.47 \text{ kN} = 10.5 \text{ kN} \qquad \textbf{Ans.}$$

Applying the law of sines to Fig. *b* and using this result, yields

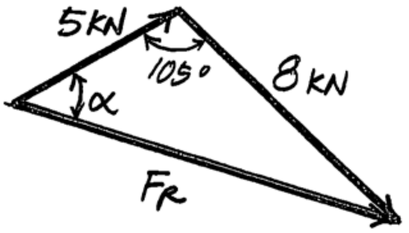
$$\frac{\sin\alpha}{8} = \frac{\sin 105°}{10.47} \qquad \alpha = 47.54°$$

Thus, the direction angle ϕ of $\mathbf{F}_R$ measured clockwise from the positive x axis is

$$\phi = \alpha - 30° = 47.54° - 30° = 17.5° \qquad \textbf{Ans.}$$

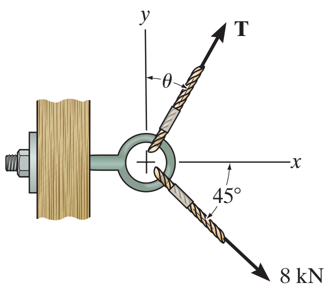


(a)



(b)

2–3. If the magnitude of the resultant force is to be 9 kN directed along the positive *x* axis, determine the magnitude of force **T** acting on the eyebolt and its angle θ.



The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of cosines to Fig. *b*,

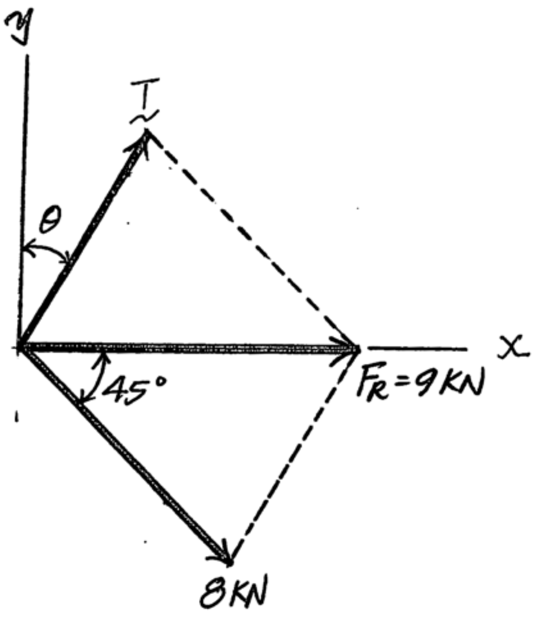
$$T = \sqrt{8^2 + 9^2 - 2(8)(9)\cos 45^\circ}$$

$$= 6.571 \text{ kN} = 6.57 \text{ kN} \qquad \textbf{Ans.}$$

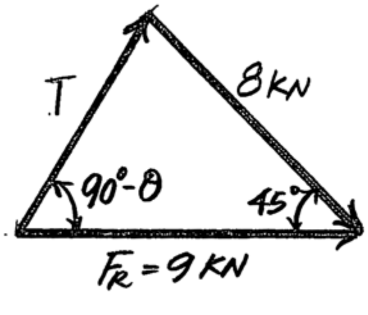
Applying the law of sines to Fig. *b* and using this result, yields

$$\frac{\sin(90^\circ - \theta)}{8} = \frac{\sin 45^\circ}{6.571}$$

$$\theta = 30.6^\circ \qquad \textbf{Ans.}$$

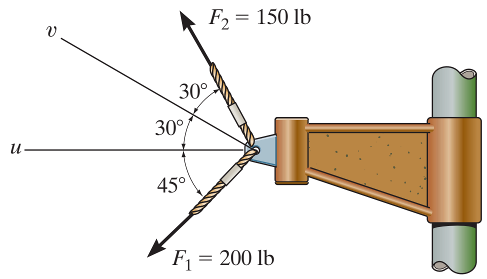


(a)



(b)

***2–4.** Determine the magnitude of the resultant force acting on the bracket and its direction measured counterclockwise from the positive *u* axis.



The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of cosines to Fig. *b*,

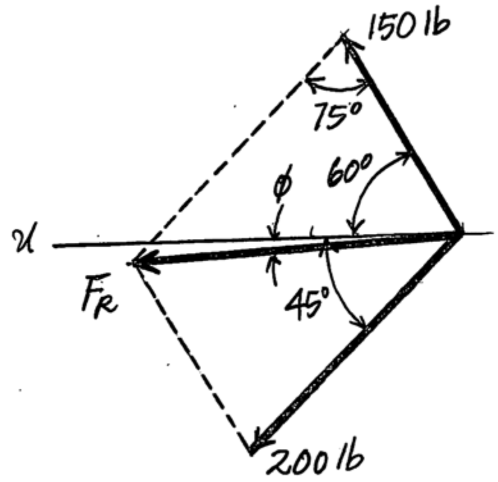
$$F_R = \sqrt{200^2 + 150^2 - 2(200)(150)\cos 75^\circ}$$
$$= 216.72 \text{ lb} = 217 \text{ lb} \qquad \textbf{Ans.}$$

Applying the law of sines to Fig. *b* and using this result yields

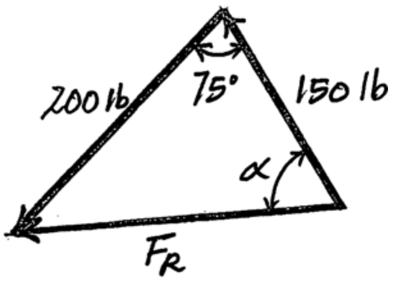
$$\frac{\sin\alpha}{200} = \frac{\sin 75^\circ}{216.72} \qquad \alpha = 63.05^\circ$$

Thus, the direction angle ϕ of $\mathbf{F}_R$, measured counterclockwise from the positive *u* axis, is

$$\phi = \alpha - 60^\circ = 63.05^\circ - 60^\circ = 3.05^\circ \qquad \textbf{Ans.}$$

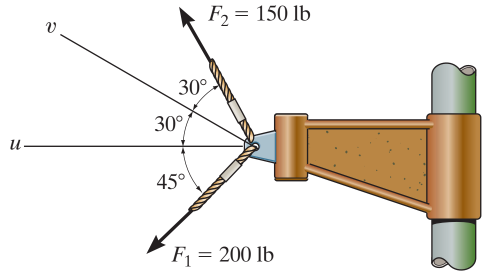


(a)



(b)

•2–5. Resolve $\mathbf{F}_1$ into components along the *u* and *v* axes, and determine the magnitudes of these components.

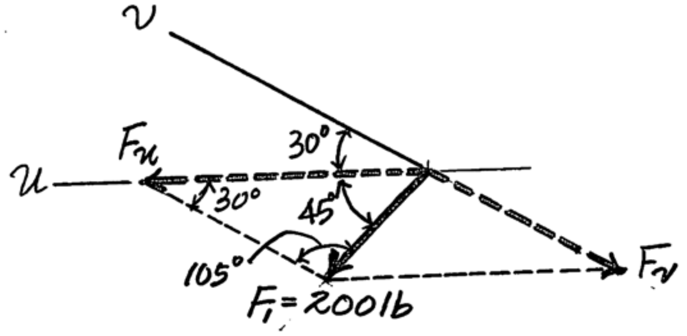


The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

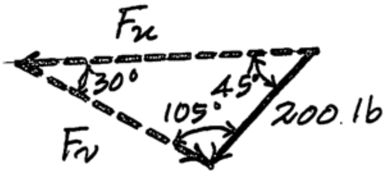
Applying the law of sines to Fig. *b*, yields

$$\frac{F_u}{\sin 105^\circ} = \frac{200}{\sin 30^\circ} \qquad F_u = 386\text{ lb}$$ **Ans.**

$$\frac{F_v}{\sin 45^\circ} = \frac{200}{\sin 30^\circ} \qquad F_v = 283\text{ lb}$$ **Ans.**

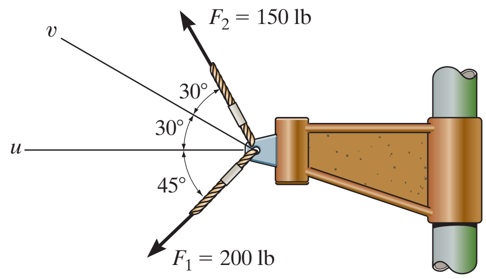


(a)



(b)

2–6. Resolve $\mathbf{F}_2$ into components along the u and v axes, and determine the magnitudes of these components.

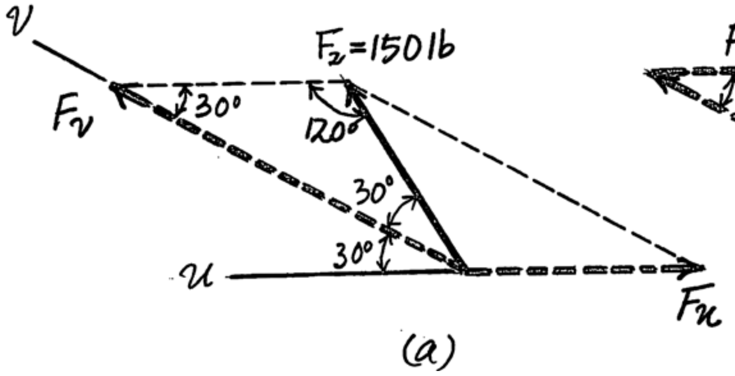


The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

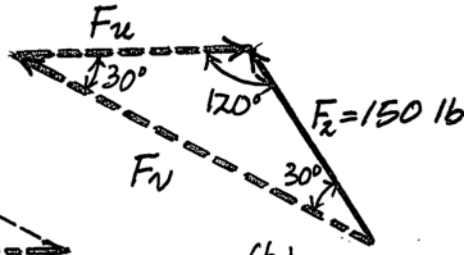
Applying the law of sines to Fig. *b*,

$$\frac{F_u}{\sin 30^\circ} = \frac{150}{\sin 30^\circ} \qquad F_u = 150 \text{ lb} \qquad \textbf{Ans.}$$

$$\frac{F_v}{\sin 120^\circ} = \frac{150}{\sin 30^\circ} \qquad F_v = 260 \text{ lb} \qquad \textbf{Ans.}$$

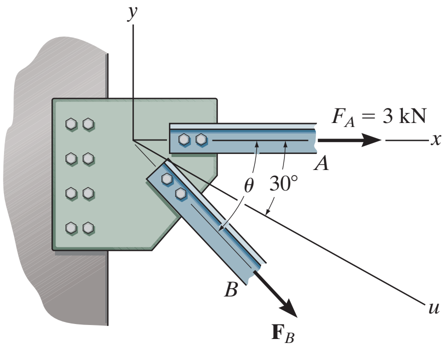


(a)



(b)

2–7. If $F_B = 2$ kN and the resultant force acts along the positive u axis, determine the magnitude of the resultant force and the angle θ.



The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of sines to Fig. *b*, yields

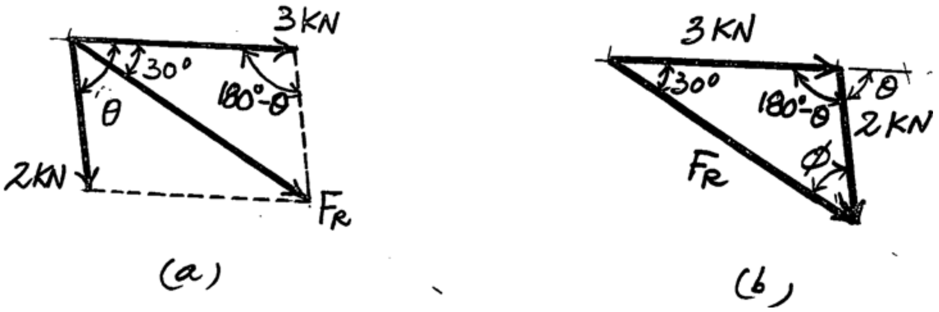
$$\frac{\sin\phi}{3} = \frac{\sin 30^\circ}{2} \qquad \phi = 48.59^\circ$$

Thus,

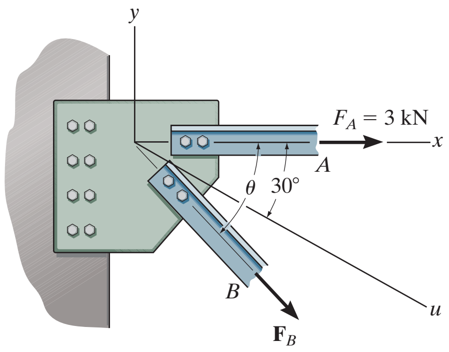
$$\theta = 30^\circ + \phi = 30^\circ + 48.59^\circ = 78.59^\circ = 78.6^\circ \qquad \textbf{Ans.}$$

With the result $\theta = 78.59^\circ$, applying the law of sines to Fig. *b* again, yields

$$\frac{F_R}{\sin(180^\circ - 78.59^\circ)} = \frac{2}{\sin 30^\circ} \qquad F_R = 3.92 \text{ kN} \qquad \textbf{Ans.}$$



***2–8.** If the resultant force is required to act along the positive u axis and have a magnitude of 5 kN, determine the required magnitude of $\mathbf{F}_B$ and its direction θ.



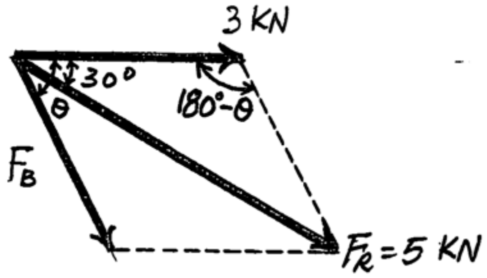
The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of cosines to Fig. *b*,

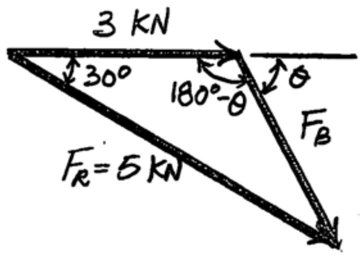
$$F_B = \sqrt{3^2 + 5^2 - 2(3)(5)\cos 30°}$$
$$= 2.832 \text{ kN} = 2.83 \text{ kN} \qquad \textbf{Ans.}$$

Using this result and realizing that $\sin(180° - \theta) = \sin\theta$, the application of the sine law to Fig. *b*, yields

$$\frac{\sin\theta}{5} = \frac{\sin 30°}{2.832} \qquad \theta = 62.0° \qquad \textbf{Ans.}$$

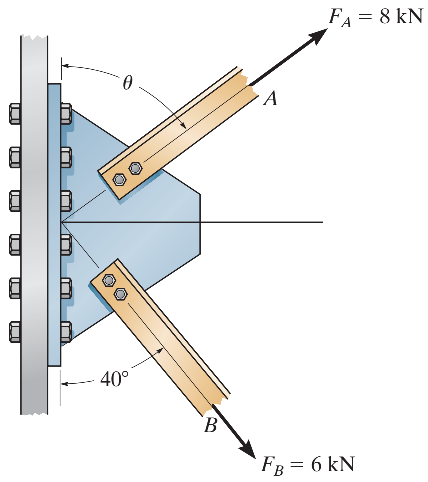


(a)



(b)

•2–9. The plate is subjected to the two forces at A and B as shown. If $\theta = 60°$, determine the magnitude of the resultant of these two forces and its direction measured clockwise from the horizontal.



Parallelogram Law : The parallelogram law of addition is shown in Fig. (a).

Trigonometry : Using law of cosines [Fig. (b)], we have

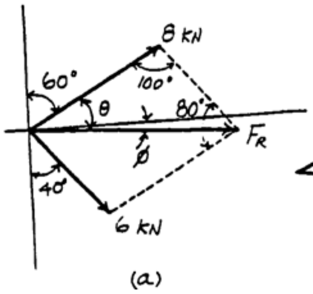
$$F_R = \sqrt{8^2 + 6^2 - 2(8)(6)\cos 100°}$$
$$= 10.80 \text{ kN} = 10.8 \text{ kN} \qquad \textbf{Ans}$$

The angle θ can be determined using law of sines [Fig. (b)].

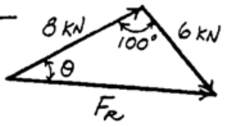
$$\frac{\sin\theta}{6} = \frac{\sin 100°}{10.80}$$
$$\sin\theta = 0.5470$$
$$\theta = 33.16°$$

Thus, the direction ϕ of $\mathbf{F}_R$ measured from the x axis is

$$\phi = 33.16° - 30° = 3.16° \qquad \textbf{Ans}$$

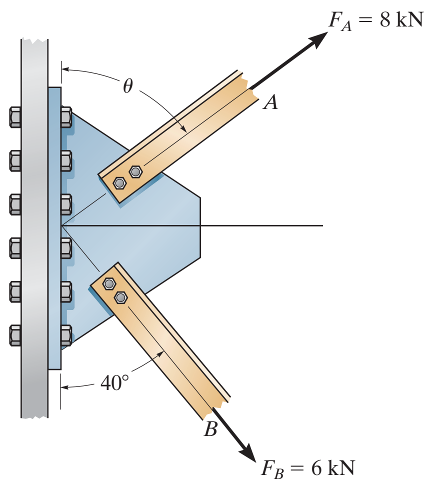


(a)



(b)

2–10. Determine the angle of θ for connecting member A to the plate so that the resultant force of $\mathbf{F}_A$ and $\mathbf{F}_B$ is directed horizontally to the right. Also, what is the magnitude of the resultant force?



Parallelogram Law : The parallelogram law of addition is shown in Fig. (a).

Trigonometry : Using law of sines [Fig. (b)], we have

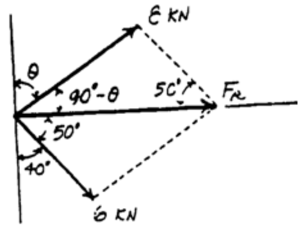
$$\frac{\sin(90° - \theta)}{6} = \frac{\sin 50°}{8}$$

$$\sin(90° - \theta) = 0.5745$$

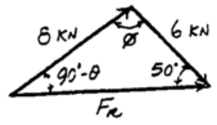
$$\theta = 54.93° = 54.9° \qquad \textbf{Ans}$$

From the triangle, $\phi = 180° - (90° - 54.93°) - 50° = 94.93°$. Thus, using law of cosines, the magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{8^2 + 6^2 - 2(8)(6)\cos 94.93°}$$
$$= 10.4 \text{ kN} \qquad \textbf{Ans}$$

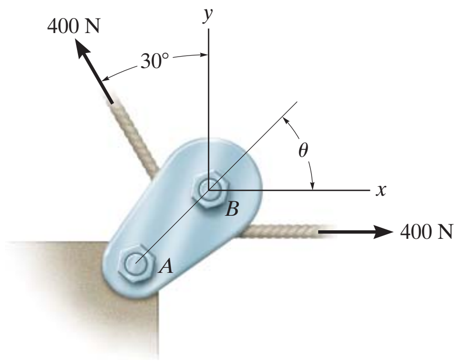


(a)



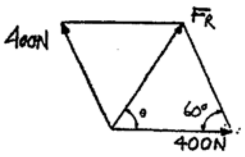
(b)

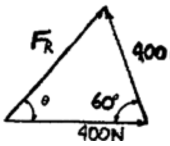
2–11. If the tension in the cable is 400 N, determine the magnitude and direction of the resultant force acting on the pulley. This angle is the same angle θ of line AB on the tailboard block.



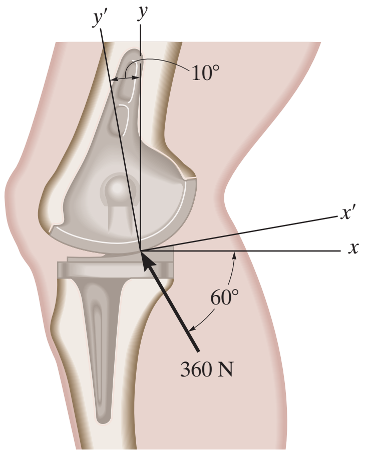
$$F_R = \sqrt{(400)^2 + (400)^2 - 2(400)(400)\cos 60°} = 400 \text{ N} \qquad \textbf{Ans}$$

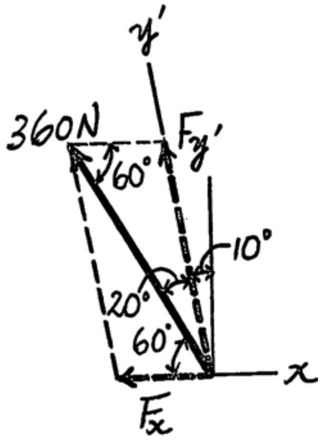
$$\frac{\sin\theta}{400} = \frac{\sin 60°}{400}; \quad \theta = 60° \qquad \textbf{Ans}$$





***2–12.** The device is used for surgical replacement of the knee joint. If the force acting along the leg is 360 N, determine its components along the x and y' axes.

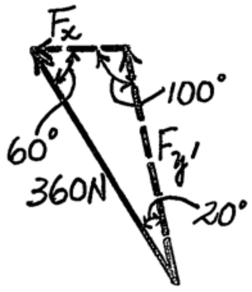




(a)

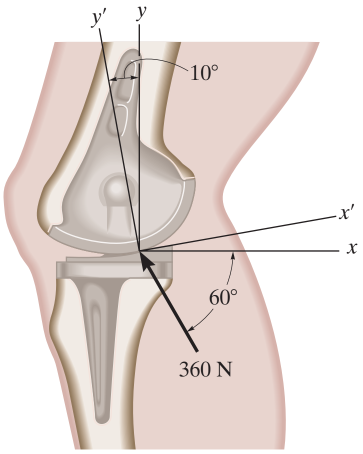
$$\frac{-F_x}{\sin 20^\circ} = \frac{360}{\sin 100^\circ}; \quad F_x = -125\text{ N} \quad \textbf{Ans}$$

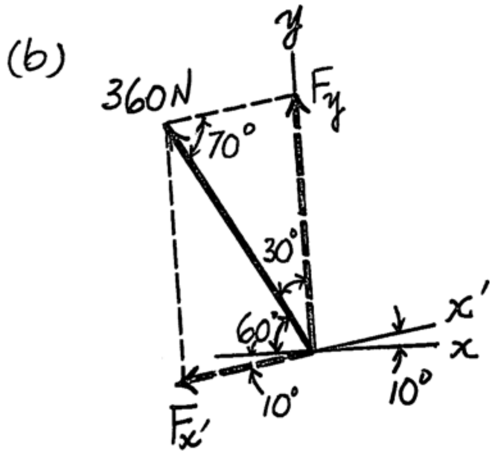
$$\frac{F_{y'}}{\sin 60^\circ} = \frac{360}{\sin 100^\circ}; \quad F_{y'} = 317\text{ N} \quad \textbf{Ans}$$



(b)

•2–13. The device is used for surgical replacement of the knee joint. If the force acting along the leg is 360 N, determine its components along the x' and y axes.

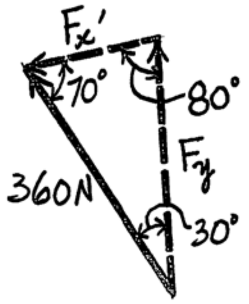




(a)

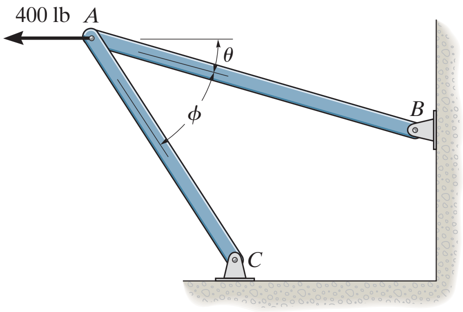
$$\frac{-F_{x'}}{\sin 30^\circ} = \frac{360}{\sin 80^\circ}; \quad F_{x'} = -183 \text{ N} \qquad \text{Ans}$$

$$\frac{F_y}{\sin 70^\circ} = \frac{360}{\sin 80^\circ}; \quad F_y = 344 \text{ N} \qquad \text{Ans}$$



(b)

2–14. Determine the design angle $\theta\ (0° \le \theta \le 90°)$ for strut AB so that the 400-lb horizontal force has a component of 500 lb directed from A towards C. What is the component of force acting along member AB? Take $\phi = 40°$.



Parallelogram Law : The parallelogram law of addition is shown in Fig. (a).

Trigonometry : Using law of sines [Fig. (b)], we have

$$\frac{\sin\theta}{500} = \frac{\sin 40°}{400}$$

$$\sin\theta = 0.8035$$

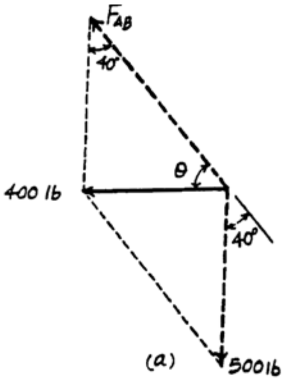
$$\theta = 53.46° = 53.5° \qquad \textbf{Ans}$$

Thus, $\psi = 180° - 40° - 53.46° = 86.54°$

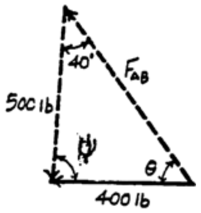
Using law of sines [Fig. (b)]

$$\frac{F_{AB}}{\sin 86.54°} = \frac{400}{\sin 40°}$$

$$F_{AB} = 621\text{ lb} \qquad \textbf{Ans}$$

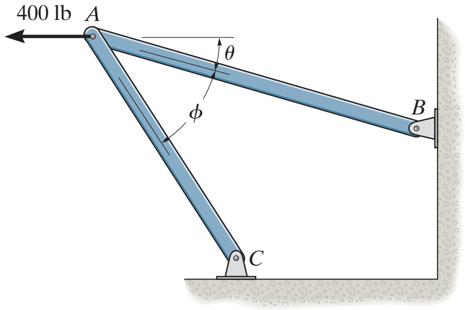


(a)



(b)

2–15. Determine the design angle $\phi\ (0° \le \phi \le 90°)$ between struts AB and AC so that the 400-lb horizontal force has a component of 600 lb which acts up to the left, in the same direction as from B towards A. Take $\theta = 30°$.



Parallelogram Law : The parallelogram law of addition is shown in Fig. (a).

Trigonometry : Using law of cosines [Fig. (b)], we have

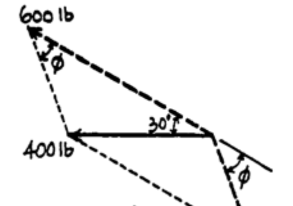
$$F_{AC} = \sqrt{400^2 + 600^2 - 2(400)(600)\cos 30°} = 322.97\text{ lb}$$

The angle ϕ can be determined using law of sines [Fig. (b)].

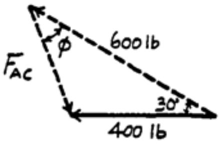
$$\frac{\sin\phi}{400} = \frac{\sin 30°}{322.97}$$

$$\sin\phi = 0.6193$$

$$\phi = 38.3° \qquad \textbf{Ans}$$

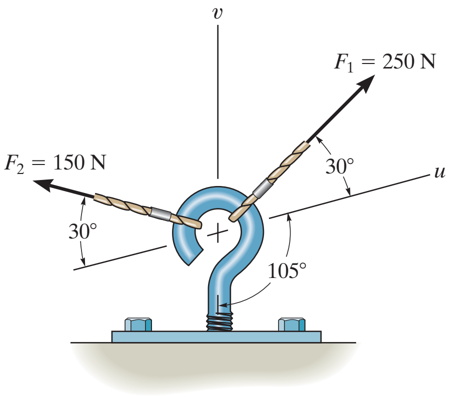


(a)



(b)

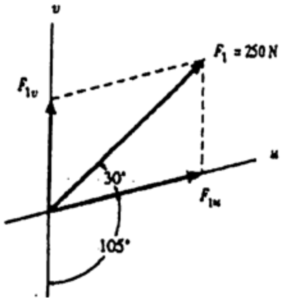
***2–16.** Resolve $\mathbf{F}_1$ into components along the u and v axes and determine the magnitudes of these components.

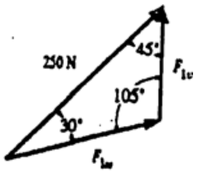


Sine law :

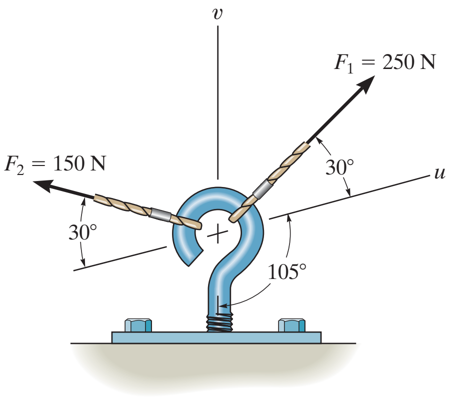
$$\frac{F_{1v}}{\sin 30°} = \frac{250}{\sin 105°} \qquad F_{1v} = 129 \text{ N} \qquad \text{Ans}$$

$$\frac{F_{1u}}{\sin 45°} = \frac{250}{\sin 105°} \qquad F_{1u} = 183 \text{ N} \qquad \text{Ans}$$





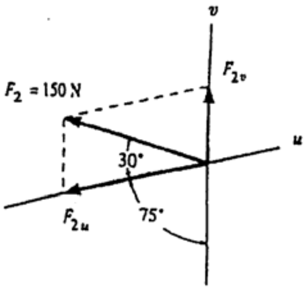
•2–17. Resolve $\mathbf{F}_2$ into components along the u and v axes and determine the magnitudes of these components.

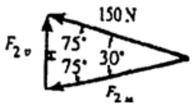


Sine law :

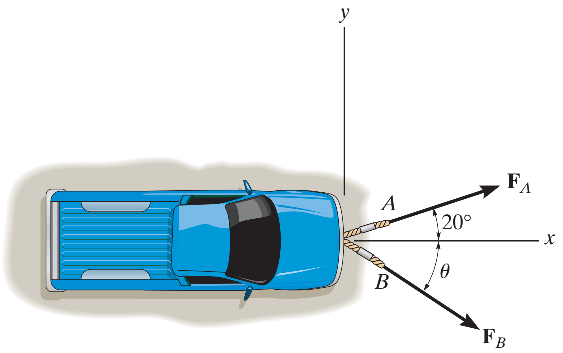
$$\frac{F_{2v}}{\sin 30°} = \frac{150}{\sin 75°} \qquad F_{2v} = 77.6 \text{ N} \qquad \text{Ans}$$

$$\frac{F_{2u}}{\sin 75°} = \frac{150}{\sin 75°} \qquad F_{2u} = 150 \text{ N} \qquad \text{Ans}$$

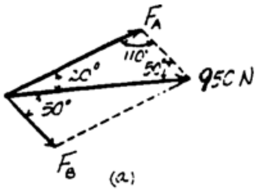




2–18. The truck is to be towed using two ropes. Determine the magnitudes of forces $\mathbf{F}_A$ and $\mathbf{F}_B$ acting on each rope in order to develop a resultant force of 950 N directed along the positive x axis. Set $\theta = 50°$.



Parallelogram Law **:** The parallelogram law of addition is shown in Fig. (a).

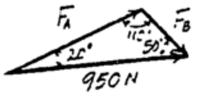


(a)

Trigonometry **:** Using law of sines [Fig. (b)], we have

$$\frac{F_A}{\sin 50°} = \frac{950}{\sin 110°}$$

$$F_A = 774 \text{ N} \qquad \textbf{Ans}$$

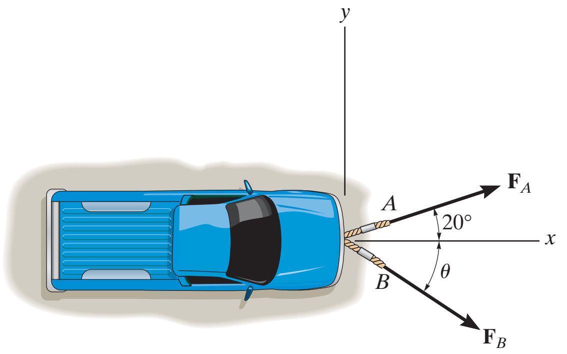


(b)

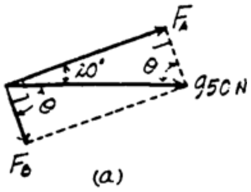
$$\frac{F_B}{\sin 20°} = \frac{950}{\sin 110°}$$

$$F_B = 346 \text{ N} \qquad \textbf{Ans}$$

2–19. The truck is to be towed using two ropes. If the resultant force is to be 950 N, directed along the positive x axis, determine the magnitudes of forces $\mathbf{F}_A$ and $\mathbf{F}_B$ acting on each rope and the angle θ of $\mathbf{F}_B$ so that the magnitude of $\mathbf{F}_B$ is a *minimum*. $\mathbf{F}_A$ acts at 20° from the x axis as shown.



Parallelogram Law **:** In order to produce a *minimum* force $\mathbf{F}_B$, $\mathbf{F}_B$ has to act perpendicular to $\mathbf{F}_A$. The parallelogram law of addition is shown in Fig. (a).

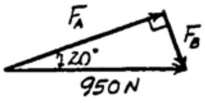


(a)

Trigonometry **:** Fig. (b).

$$F_B = 950 \sin 20° = 325 \text{ N} \qquad \textbf{Ans}$$

$$F_A = 950 \cos 20° = 893 \text{ N} \qquad \textbf{Ans}$$

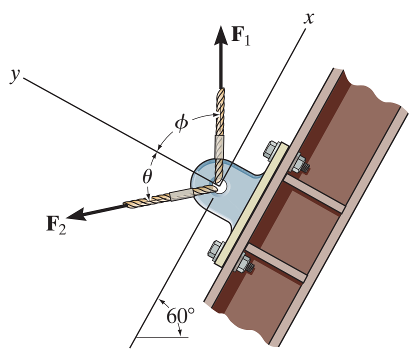


(b)

The angle θ is

$$\theta = 90° - 20° = 70.0° \qquad \textbf{Ans}$$

***2–20.** If $\phi = 45°$, $F_1 = 5$ kN, and the resultant force is 6 kN directed along the positive y axis, determine the required magnitude of $\mathbf{F}_2$ and its direction θ.



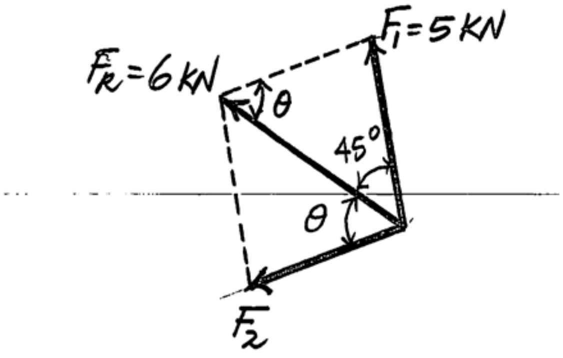
The parallelogram law of addition and triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of cosines to Fig. *b*,

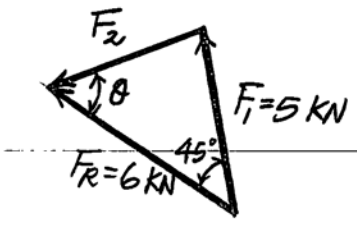
$$F_2 = \sqrt{6^2 + 5^2 - 2(6)(5)\cos 45°}$$
$$= 4.310 \text{ kN} = 4.31 \text{ kN} \qquad \textbf{Ans.}$$

Using this result and applying the law of sines to Fig. *b*, yields

$$\frac{\sin\theta}{5} = \frac{\sin 45°}{4.310} \qquad \theta = 55.1° \quad \textbf{Ans.}$$

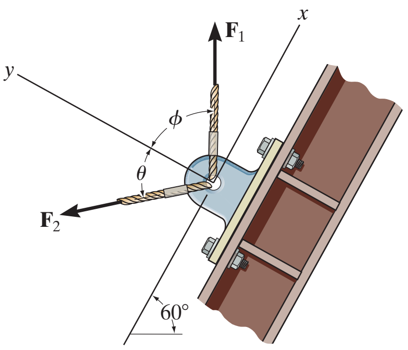


(a)



(b)

•2–21. If $\phi = 30^\circ$ and the resultant force is to be 6 kN directed along the positive y axis, determine the magnitudes of $\mathbf{F}_1$ and $\mathbf{F}_2$ and the angle θ if F_2 is required to be a minimum.



For $\mathbf{F}_2$ to be minimum, it has to be directed perpendicular to F_R.

The parallelogram law of addition and triangular rule are shown in Figs. *a* and *b*, respectively.

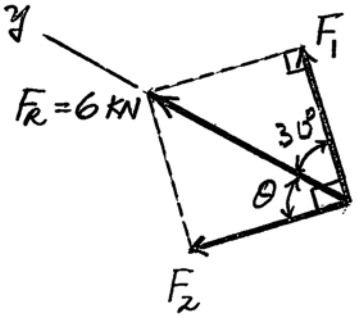
By applying simple trigonometry to Fig. *b*,

$$F_1 = 6\cos 30^\circ = 5.20 \text{ kN} \qquad \textbf{Ans.}$$

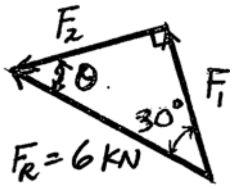
$$F_2 = 6\sin 30^\circ = 3 \text{ kN} \qquad \textbf{Ans.}$$

and

$$\theta = 90^\circ - 30^\circ = 60^\circ \quad \textbf{Ans.}$$

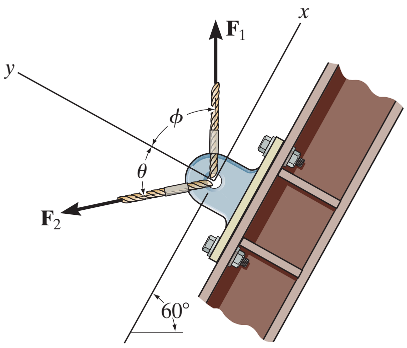


(a)



(b)

2–22. If $\phi = 30°$, $F_1 = 5$ kN, and the resultant force is to be directed along the positive y axis, determine the magnitude of the resultant force if F_2 is to be a minimum. Also, what is F_2 and the angle θ?



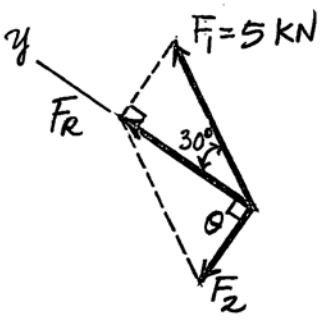
Parallelogram Law and Triangular Rule: The parallelogram law of addition and triangular rule are shown in Figs. *a* and *b*, respectively.

For $\mathbf{F}_2$ to be minimum, it must be directed perpendicular to the resultant force. Thus,

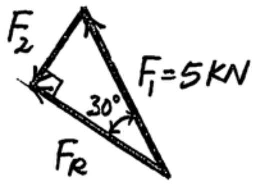
$$\theta = 90° \qquad \textbf{Ans.}$$

By applying simple trigonometry to Fig. *b*,

$$F_2 = 5 \sin 30° = 2.50 \text{ kN} \qquad \textbf{Ans.}$$
$$F_R = 5 \cos 30° = 4.33 \text{ kN} \qquad \textbf{Ans.}$$

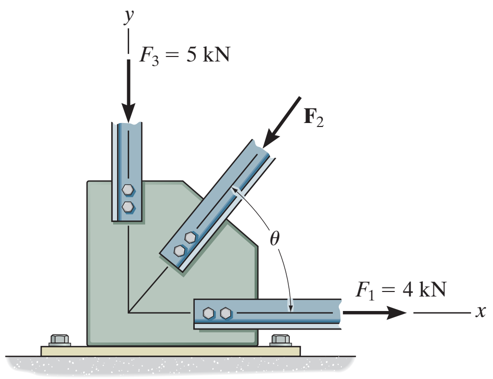


(a)



(b)

2–23. If $\theta = 30°$ and $F_2 = 6$ kN, determine the magnitude of the resultant force acting on the plate and its direction measured clockwise from the positive x axis.



Parallelogram Law and Triangular Rule: This problem can be solved by adding the forces successively, using the parallelogram law of addition, shown in Fig. *a*. Two triangular force diagrams, shown in Figs. *b* and *c*, can be derived from the parallelogram.

Determination of Unknowns: Referring to Fig. *b*, F' and α can be determined as follows.

$$F' = \sqrt{4^2 + 5^2} = 6.403 \text{ kN}$$

$$\tan\alpha = \frac{5}{4} \qquad \alpha = 51.34°$$

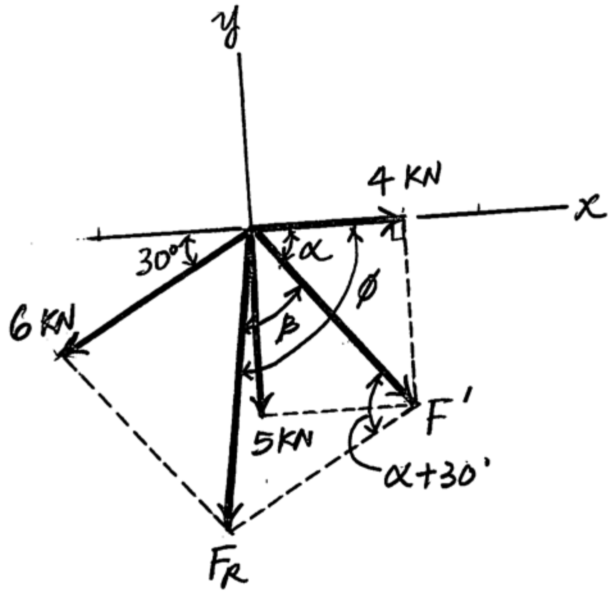
Using the results for F' and α and referring to Fig. *c*, $\mathbf{F}_R$ and β can be determined.

$$F_R = \sqrt{6^2 + 6.403^2 - 2(6)(6.403)\cos(51.34° + 30°)}$$
$$= 8.089 \text{ kN} = 8.09 \text{ kN} \qquad \textbf{Ans.}$$

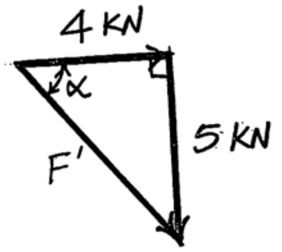
$$\frac{\sin\beta}{6} = \frac{\sin(51.34° + 30°)}{8.089} \qquad \beta = 47.16°$$

Thus, the direction angle ϕ of $\mathbf{F}_R$, measured clockwise from the positive x axis, is

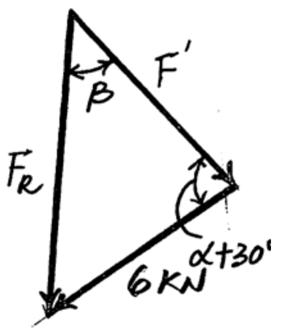
$$\phi = \alpha + \beta = 51.34° + 47.16° = 98.5° \qquad \textbf{Ans.}$$



(a)

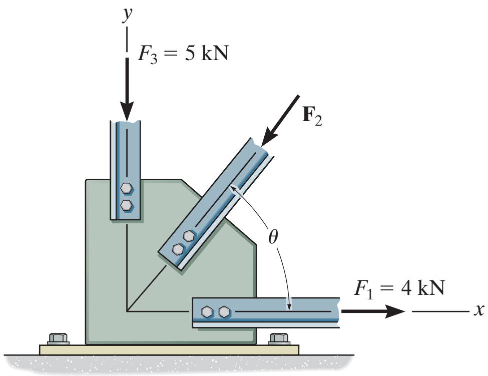


(b)



(c)

***2–24.** If the resultant force $\mathbf{F}_R$ is directed along a line measured 75° clockwise from the positive x axis and the magnitude of $\mathbf{F}_2$ is to be a minimum, determine the magnitudes of $\mathbf{F}_R$ and $\mathbf{F}_2$ and the angle $\theta \leq 90°$.



This problem can be solved by adding the forces successively, using the parallelogram law of addition, shown in Fig. *a*. Two triangular force diagrams, shown in Figs. *b* and *c*, can be derived from the parallelograms. For $\mathbf{F}_1$ to be minimum, it must be perpendicular to the resultant force' s line of action. Thus,

$$\theta = 90° - 75° = 15° \qquad \textbf{Ans.}$$

Referring to Fig. *b*, F' and α can be determined.

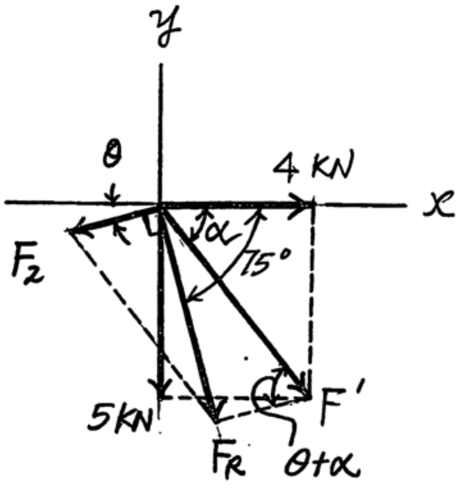
$$F' = \sqrt{4^2 + 5^2} = 6.403 \text{ kN}$$

$$\tan\alpha = \frac{5}{4} \qquad \alpha = 51.34°$$

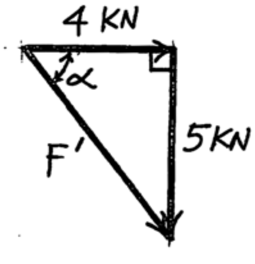
Using the results for θ, α, and F', $\mathbf{F}_R$ and $\mathbf{F}_2$ can be determined by referring to Fig. *c*.

$$F_2 = 6.403\cos(15° + 51.43°) = 2.57 \text{ kN} \qquad \textbf{Ans.}$$

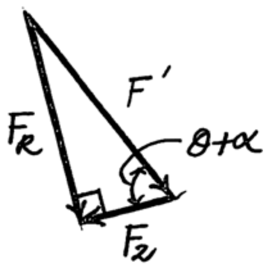
$$F_R = 6.403\sin(15° + 51.43°) = 5.86 \text{ kN} \qquad \textbf{Ans.}$$



(a)

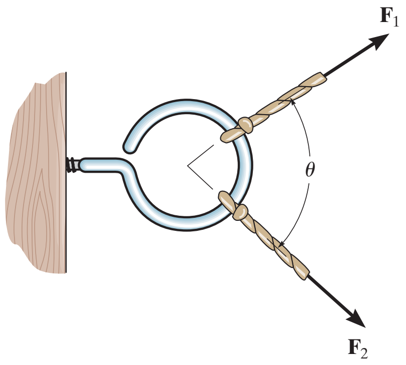


(b)



(c)

•2–25. Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on the screw eye. If their lines of action are at an angle θ apart and the magnitude of each force is $F_1 = F_2 = F$, determine the magnitude of the resultant force $\mathbf{F}_R$ and the angle between $\mathbf{F}_R$ and $\mathbf{F}_1$.



$$\frac{F}{\sin\phi} = \frac{F}{\sin(\theta - \phi)}$$

$$\sin(\theta - \phi) = \sin\phi$$

$$\theta - \phi = \phi$$

$$\phi = \frac{\theta}{2} \qquad \textbf{Ans}$$

$$F_R = \sqrt{(F)^2 + (F)^2 - 2(F)(F)\cos(180^\circ - \theta)}$$

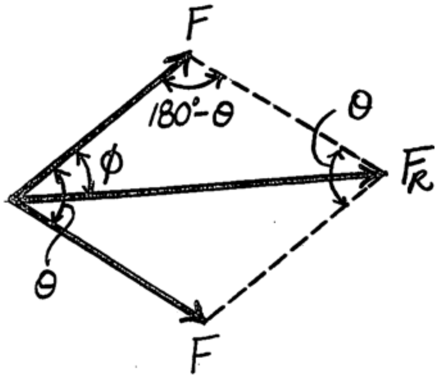
Since $\cos(180^\circ - \theta) = -\cos\theta$

$$F_R = F(\sqrt{2})\sqrt{1 + \cos\theta}$$

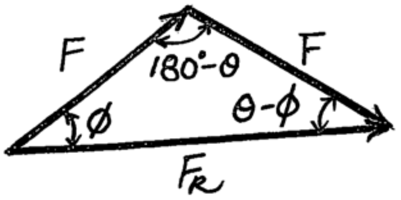
Since $\cos(\frac{\theta}{2}) = \sqrt{\frac{1 + \cos\theta}{2}}$

Then

$$F_R = 2F\cos(\frac{\theta}{2}) \qquad \textbf{Ans}$$

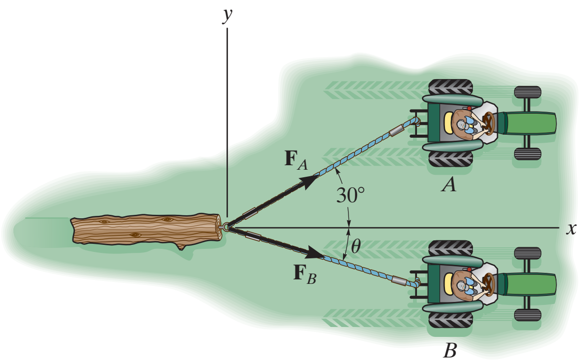


(a)



(b)

2–26. The log is being towed by two tractors A and B. Determine the magnitudes of the two towing forces $\mathbf{F}_A$ and $\mathbf{F}_B$ if it is required that the resultant force have a magnitude $F_R = 10$ kN and be directed along the x axis. Set $\theta = 15°$.



Parallelogram Law **:** The parallelogram law of addition is shown in Fig. (a).

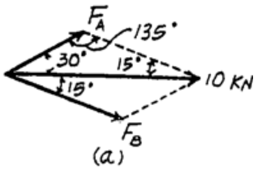
Trigonometry **:** Using law of sines [Fig. (b)], we have

$$\frac{F_A}{\sin 15°} = \frac{10}{\sin 135°}$$

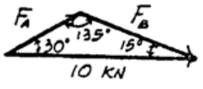
$$F_A = 3.66 \text{ kN} \qquad \textbf{Ans}$$

$$\frac{F_B}{\sin 30°} = \frac{10}{\sin 135°}$$

$$F_B = 7.07 \text{ kN} \qquad \textbf{Ans}$$

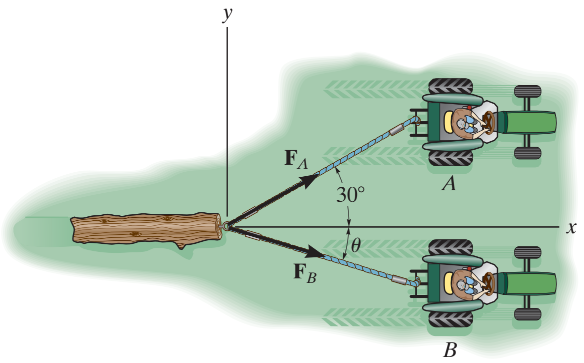


(a)



(b)

2–27. The resultant $\mathbf{F}_R$ of the two forces acting on the log is to be directed along the positive x axis and have a magnitude of 10 kN, determine the angle θ of the cable, attached to B such that the magnitude of force $\mathbf{F}_B$ in this cable is a minimum. What is the magnitude of the force in each cable for this situation?



Parallelogram Law **:** In order to produce a *minimum* force $\mathbf{F}_B$, $\mathbf{F}_B$ has to act perpendicular to $\mathbf{F}_A$. The parallelogram law of addition is shown in Fig. (a).

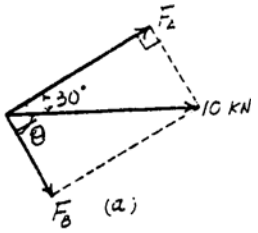
Trigonometry **:** Fig. (b).

$$F_B = 10 \sin 30° = 5.00 \text{ kN} \qquad \textbf{Ans}$$

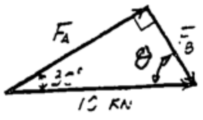
$$F_A = 10 \cos 30° = 8.66 \text{ kN} \qquad \textbf{Ans}$$

The angle θ is

$$\theta = 90° - 30° = 60.0° \qquad \textbf{Ans}$$

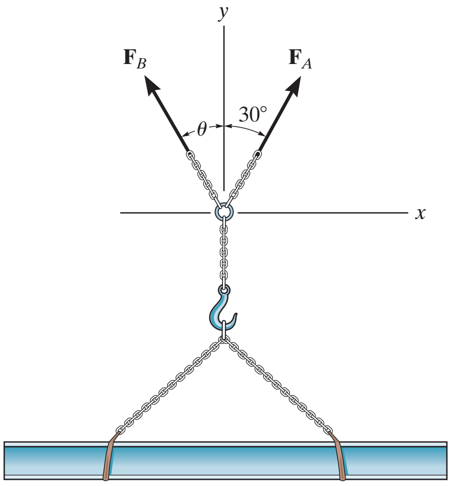


(a)



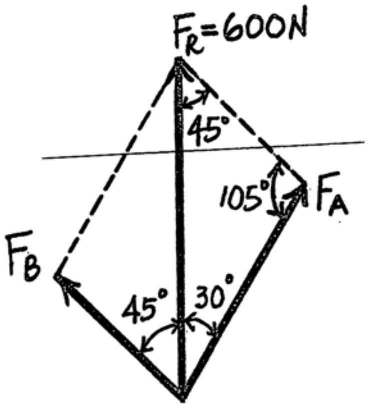
(b)

***2–28.** The beam is to be hoisted using two chains. Determine the magnitudes of forces $\mathbf{F}_A$ and $\mathbf{F}_B$ acting on each chain in order to develop a resultant force of 600 N directed along the positive y axis. Set $\theta = 45°$.

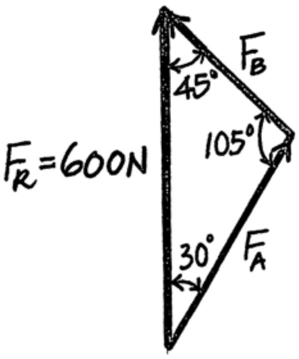


$\frac{F_A}{\sin 45°} = \frac{600}{\sin 105°}$; $F_A = 439$ N Ans

$\frac{F_B}{\sin 30°} = \frac{600}{\sin 105°}$; $F_B = 311$ N Ans

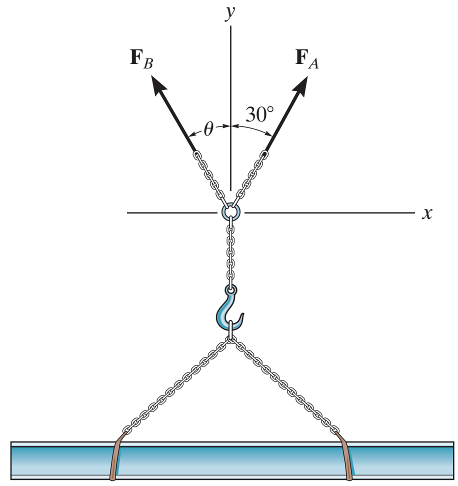


(a)



(b)

•2–29. The beam is to be hoisted using two chains. If the resultant force is to be 600 N directed along the positive y axis, determine the magnitudes of forces $\mathbf{F}_A$ and $\mathbf{F}_B$ acting on each chain and the angle θ of $\mathbf{F}_B$ so that the magnitude of $\mathbf{F}_B$ is a *minimum*. $\mathbf{F}_A$ acts at 30° from the y axis, as shown.

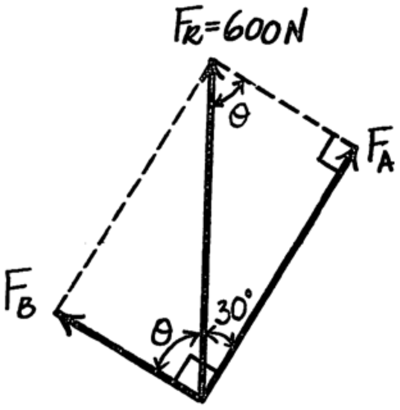


For minimum F_B, require

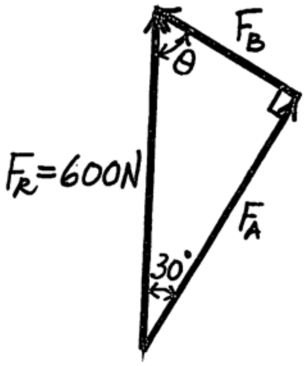
$\theta = 60°$ Ans

$F_A = 600 \cos 30° = 520\text{ N}$ Ans

$F_B = 600 \sin 30° = 300\text{ N}$ Ans

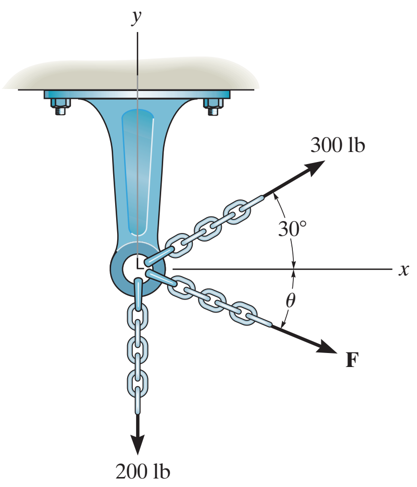


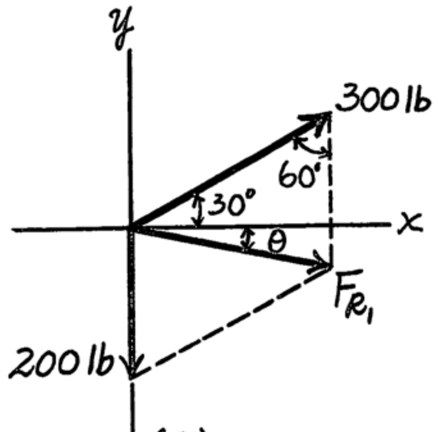
(a)



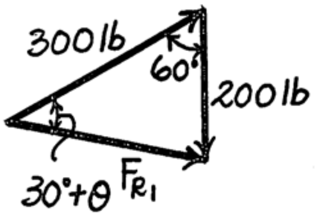
(b)

2–30. Three chains act on the bracket such that they create a resultant force having a magnitude of 500 lb. If two of the chains are subjected to known forces, as shown, determine the angle θ of the third chain measured clockwise from the positive x axis, so that the magnitude of force **F** in this chain is a *minimum*. All forces lie in the x–y plane. What is the magnitude of **F**? *Hint*: First find the resultant of the two known forces. Force **F** acts in this direction.

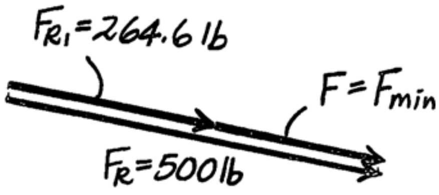




(a)



(b)



(c)

Cosine law :

$$F_{R1} = \sqrt{300^2 + 200^2 - 2(300)(200)\cos 60^\circ} = 264.6\text{ lb}$$

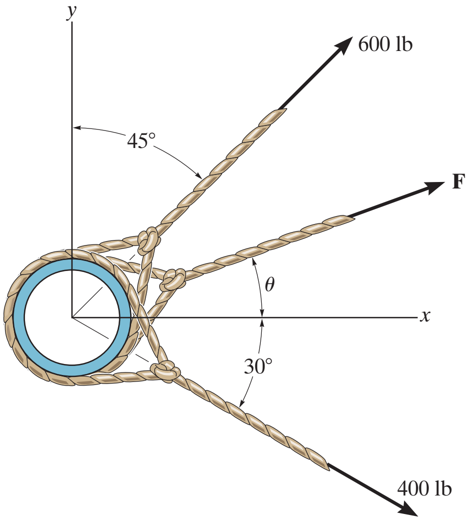
Sine law :

$$\frac{\sin(30^\circ + \theta)}{200} = \frac{\sin 60^\circ}{264.6} \qquad \theta = 10.9^\circ \qquad \text{Ans}$$

When F is directed along F_{R1} , F will be minimum to create the resultant force.

$$F_R = F_{R1} + F$$
$$500 = 264.6 + F_{min}$$
$$F_{min} = 235\text{ lb} \qquad \text{Ans}$$

2–31. Three cables pull on the pipe such that they create a resultant force having a magnitude of 900 lb. If two of the cables are subjected to known forces, as shown in the figure, determine the angle θ of the third cable so that the magnitude of force **F** in this cable is a *minimum*. All forces lie in the x–y plane. What is the magnitude of **F**? *Hint*: First find the resultant of the two known forces.

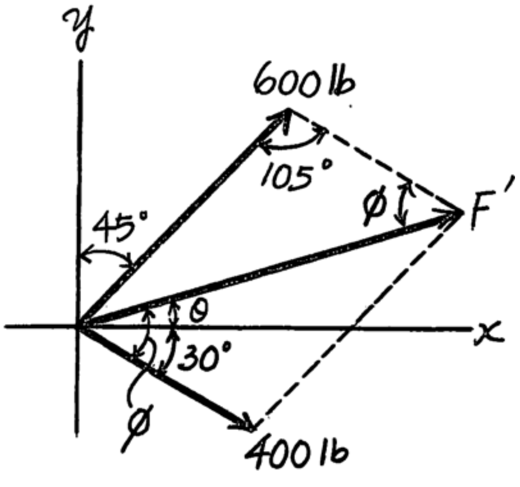


$$F' = \sqrt{(600)^2 + (400)^2 - 2(600)(400)\cos 105^\circ} = 802.64 \text{ lb}$$

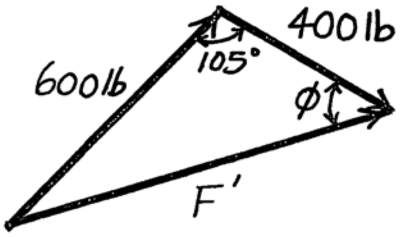
$$F = 900 - 802.64 = 97.4 \text{ lb} \qquad \textbf{Ans}$$

$$\frac{\sin\phi}{600} = \frac{\sin 105^\circ}{802.64}; \quad \phi = 46.22^\circ$$

$$\theta = 46.22^\circ - 30^\circ = 16.2^\circ \qquad \textbf{Ans}$$



(a)



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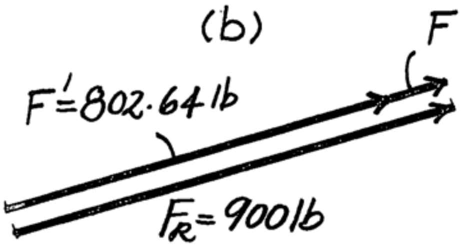
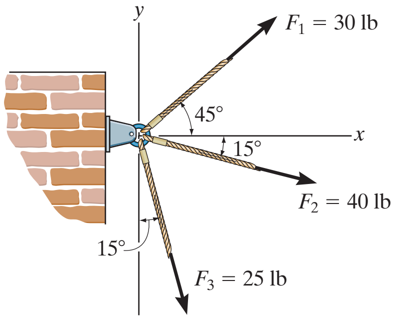


(c)

***2–32.** Determine the magnitude of the resultant force acting on the pin and its direction measured clockwise from the positive *x* axis.



Rectangular Components: By referring to Fig. *a*, the *x* and *y* components of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ can be written as

$$(F_1)_x = 30\cos 45° = 21.21\text{ lb} \qquad (F_1)_y = 30\sin 45° = 21.21\text{ lb}$$
$$(F_2)_x = 40\cos 15° = 38.64\text{ lb} \qquad (F_2)_y = 40\sin 15° = 10.35\text{ lb}$$
$$(F_3)_x = 25\sin 15° = 6.47\text{ lb} \qquad (F_3)_y = 25\cos 15° = 24.15\text{ lb}$$

Resultant Force: Summing the force components algebraically along the *x* and *y* axes,

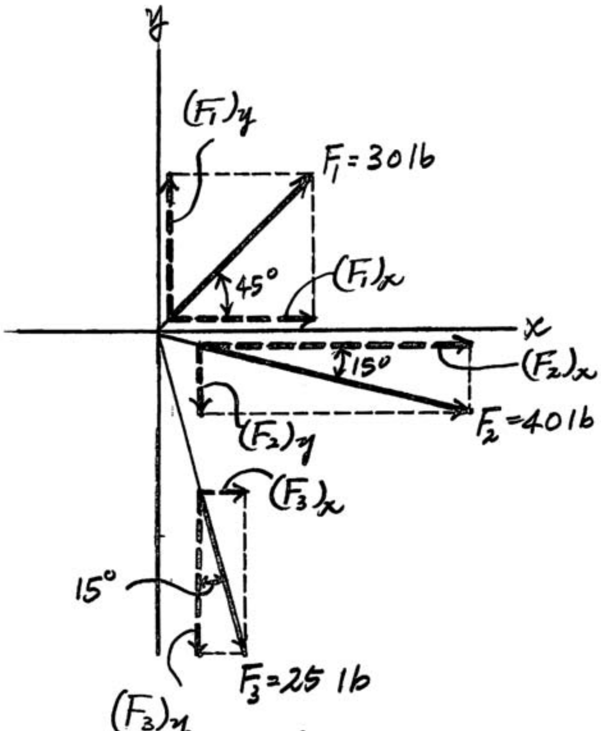
$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \quad (F_R)_x = 21.21 + 38.64 + 6.47 = 66.32\text{ lb} \rightarrow$$
$$+\uparrow\Sigma(F_R)_y = \Sigma F_y; \quad (F_R)_y = 21.21 - 10.35 - 24.15 = -13.29\text{ lb} = 13.29\text{ lb} \downarrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is

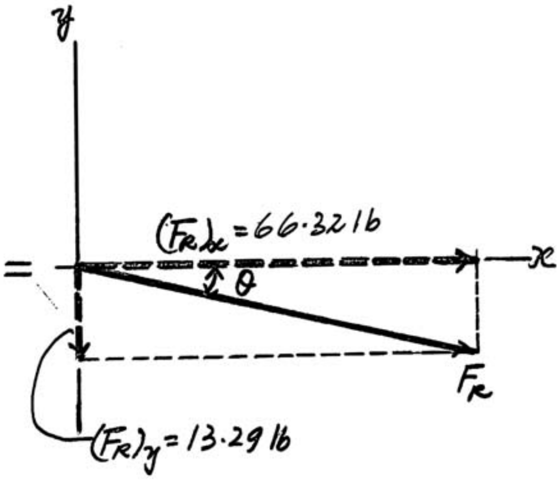
$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2} = \sqrt{66.32^2 + 13.29^2} = 67.6\text{ lb} \qquad \textbf{Ans.}$$

The direction angle θ of $\mathbf{F}_R$, measured clockwise from the positive *x* axis, is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{13.29}{66.32}\right) = 11.3° \qquad \textbf{Ans.}$$

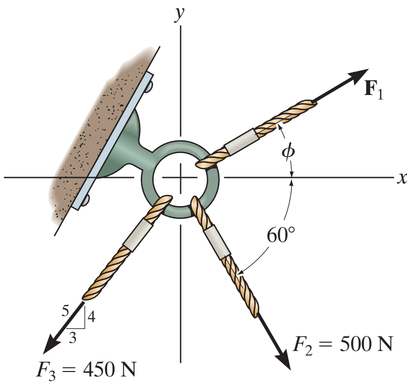


(a)



(b)

•2–33. If $F_1 = 600\text{ N}$ and $\phi = 30°$, determine the magnitude of the resultant force acting on the eyebolt and its direction measured clockwise from the positive x axis.



Rectangular Components: By referring to Fig. a, the x and y components of each force can be written as

$$(F_1)_x = 600\cos 30° = 519.62\text{ N} \qquad (F_1)_y = 600\sin 30° = 300\text{ N}$$

$$(F_2)_x = 500\cos 60° = 250\text{ N} \qquad (F_2)_y = 500\sin 60° = 433.0\text{ N}$$

$$(F_3)_x = 450\left(\frac{3}{5}\right) = 270\text{ N} \qquad (F_3)_y = 450\left(\frac{4}{5}\right) = 360\text{ N}$$

Resultant Force: Summing the force components algebraically along the x and y axes,

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \quad (F_R)_x = 519.62 + 250 - 270 = 499.62\text{ N} \rightarrow$$

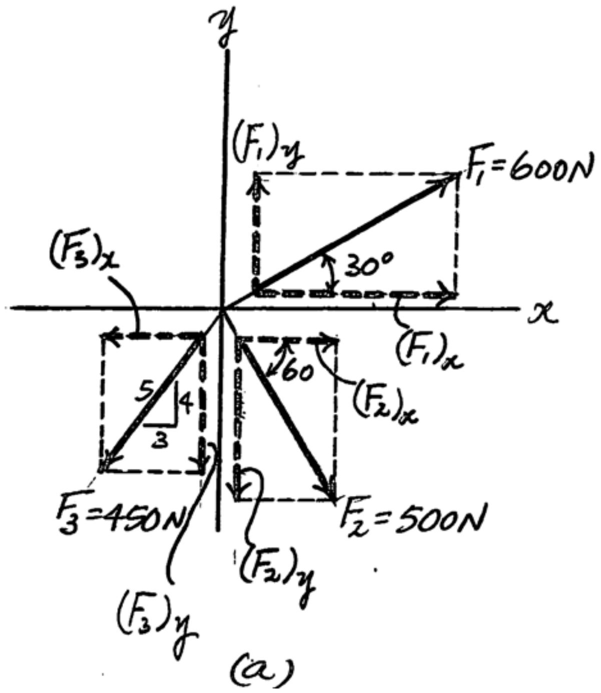
$$+\uparrow\Sigma(F_R)_y = \Sigma F_y; \quad (F_R)_y = 300 - 433.01 - 360 = -493.01\text{N} = 493.01\text{ N} \downarrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is

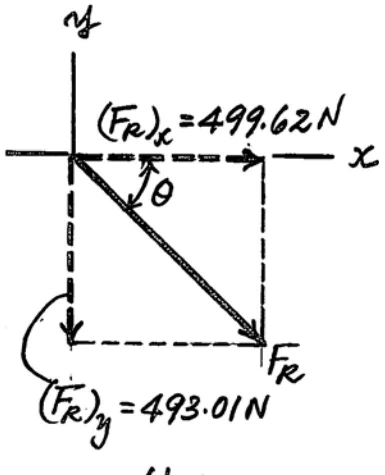
$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2} = \sqrt{499.62^2 + 493.01^2} = 701.91\text{ N} = 702\text{ N}$$ **Ans.**

The direction angle θ of $\mathbf{F}_R$, Fig. b, measured clockwise from the x axis, is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{493.01}{499.62}\right) = 44.6°$$ **Ans.**

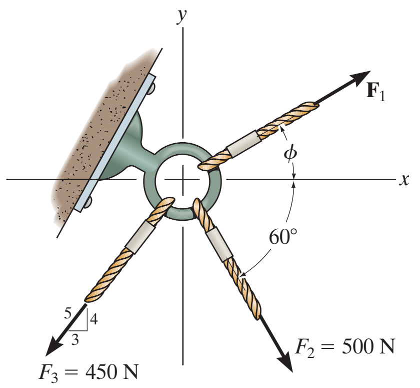


(a)



(b)

2–34. If the magnitude of the resultant force acting on the eyebolt is 600 N and its direction measured clockwise from the positive x axis is $\theta = 30°$, determine the magnitude of $\mathbf{F}_1$ and the angle ϕ.



Rectangular Components: By referring to Figs. *a* and *b*, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, and $\mathbf{F}_R$ can be written as

$$(F_1)_x = F_1\cos\phi \qquad (F_1)_y = F_1\sin\phi$$

$$(F_2)_x = 500\cos 60° = 250\text{ N} \qquad (F_2)_y = 500\sin 60° = 433.01\text{ N}$$

$$(F_3)_x = 450\left(\frac{3}{5}\right) = 270\text{ N} \qquad (F_3)_y = 450\left(\frac{4}{5}\right) = 360\text{ N}$$

$$(F_R)_x = 600\cos 30° = 519.62\text{ N} \qquad (F_R)_y = 600\sin 30° = 300\text{ N}$$

Resultant Force: Summing the force components algebraically along the x and y axes,

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \quad 519.62 = F_1\cos\phi + 250 - 270$$

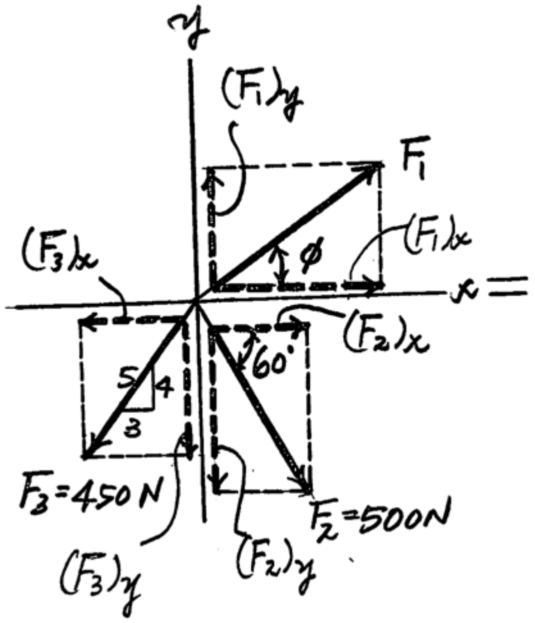
$$F_1\cos\phi = 539.62 \qquad (1)$$

$$+\uparrow\Sigma(F_R)_y = \Sigma F_y; \quad -300 = F_1\sin\phi - 433.01 - 360$$

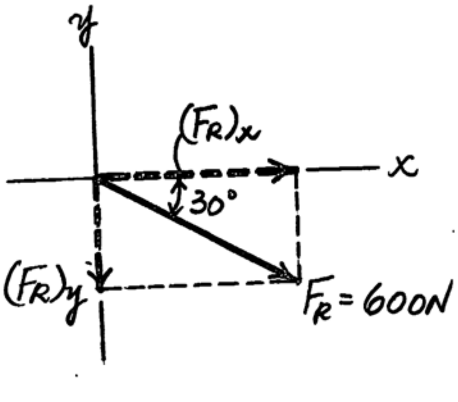
$$F_1\sin\phi = 493.01 \qquad (2)$$

Solving Eqs. (1) and (2), yields

$\phi = 42.4°$ $\qquad F_1 = 731\text{ N}$ $\qquad$ **Ans.**

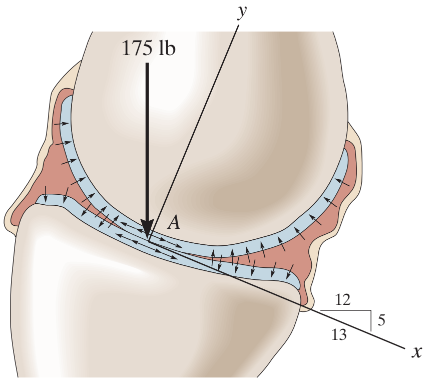


(a)



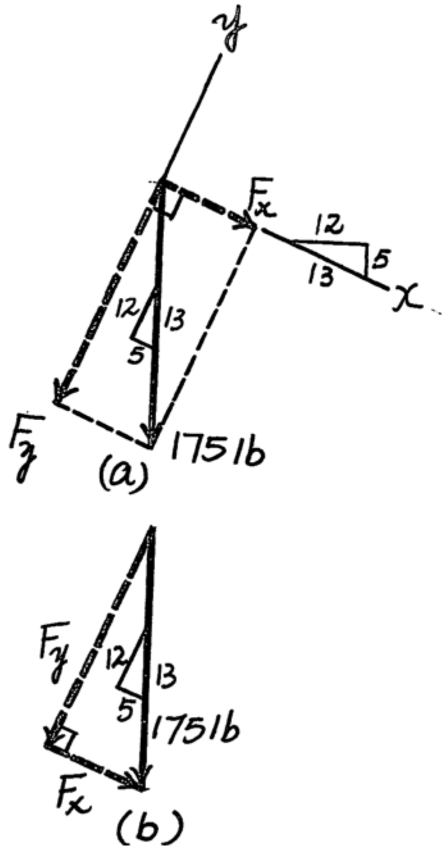
(b)

2–35. The contact point between the femur and tibia bones of the leg is at *A*. If a vertical force of 175 lb is applied at this point, determine the components along the *x* and *y* axes. Note that the *y* component represents the normal force on the load-bearing region of the bones. Both the *x* and *y* components of this force cause synovial fluid to be squeezed out of the bearing space.

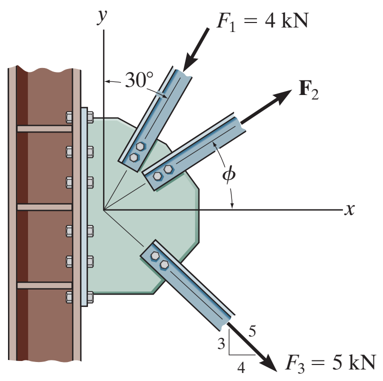


$$F_x = 175\left(\frac{5}{13}\right) = 67.3\text{ lb} \qquad \text{Ans}$$

$$F_y = -175\left(\frac{12}{13}\right) = -162\text{ lb} \qquad \text{Ans}$$



***2–36.** If $\phi = 30°$ and $F_2 = 3$ kN, determine the magnitude of the resultant force acting on the plate and its direction θ measured clockwise from the positive x axis.



Rectangular Components: By referring to Fig. *a*, the *x* and *y* components of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ can be written as

$$(F_1)_x = 4\sin 30° = 2\text{ kN} \qquad (F_1)_y = 4\cos 30° = 3.464\text{ kN}$$

$$(F_2)_x = 3\cos 30° = 2.598\text{ kN} \qquad (F_2)_y = 3\sin 30° = 1.50\text{ kN}$$

$$(F_3)_x = 5\left(\frac{4}{5}\right) = 4\text{ kN} \qquad (F_3)_y = 5\left(\frac{3}{5}\right) = 3\text{kN}$$

Resultant Force: Summing the force components algebraically along the *x* and *y* axes,

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \quad (F_R)_x = -2 + 2.598 + 4 = 4.598\text{ kN} \rightarrow$$

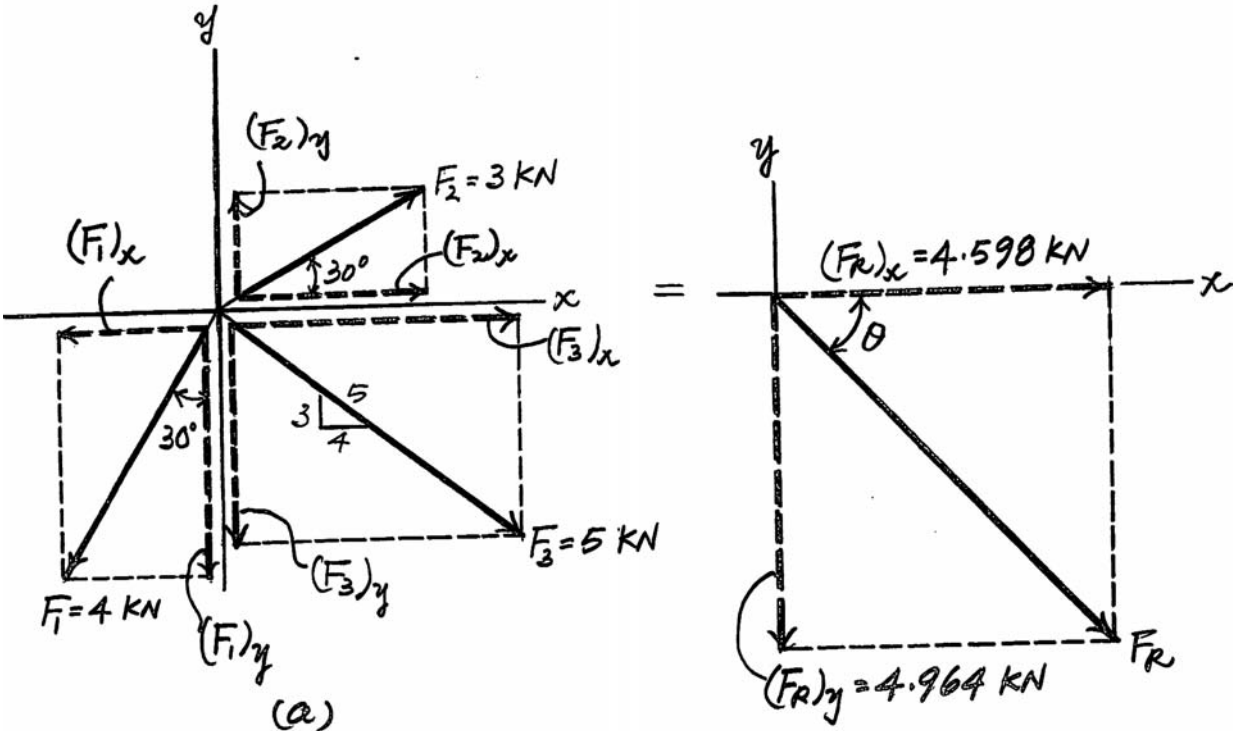
$$+\uparrow\Sigma(F_R)_y = \Sigma F_y; \quad (F_R)_y = -3.464 + 1.50 - 3 = -4.964\text{ kN} = 4.964\text{ kN} \downarrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2} = \sqrt{4.598^2 + 4.964^2} = 6.77\text{ kN} \qquad \textbf{Ans.}$$

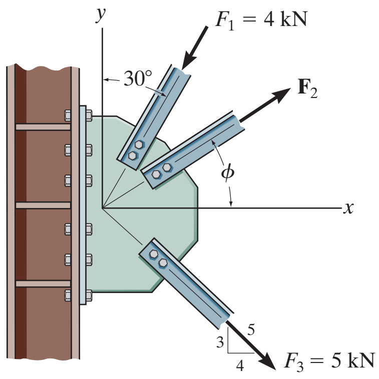
The direction angle θ of $\mathbf{F}_R$, Fig. *b*, measured clockwise from the positive *x* axis, is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{4.964}{4.598}\right) = 47.2° \qquad \textbf{Ans.}$$



(a)

•2–37. If the magnitude for the resultant force acting on the plate is required to be 6 kN and its direction measured clockwise from the positive x axis is $\theta = 30°$, determine the magnitude of $\mathbf{F}_2$ and its direction ϕ.



Rectangular Components: By referring to Figs. a and b, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, and $\mathbf{F}_R$ can be written as

$$(F_1)_x = 4\sin 30° = 2\text{ kN} \qquad (F_1)_y = 4\cos 30° = 3.464\text{ kN}$$

$$(F_2)_x = F_2\cos\phi \qquad (F_2)_y = F_2\sin\phi$$

$$(F_3)_x = 5\left(\frac{4}{5}\right) = 4\text{ kN} \qquad (F_3)_y = 5\left(\frac{3}{5}\right) = 3\text{ kN}$$

$$(F_R)_x = 6\cos 30° = 5.196\text{ kN} \qquad (F_R)_y = 6\sin 30° = 3\text{ kN}$$

Resultant Force: Summing the force components algebraically along the x and y axes,

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \quad 5.196 = -2 + F_2\cos\phi + 4$$

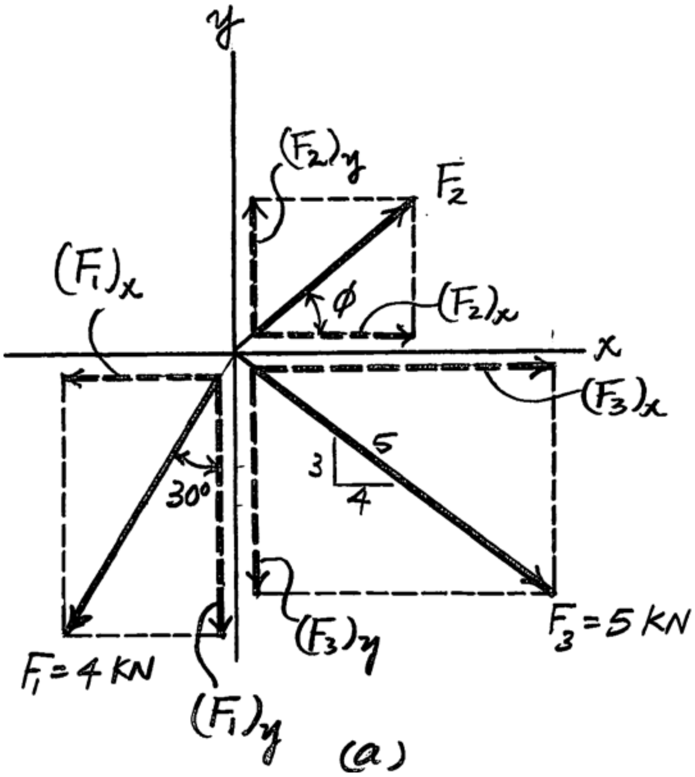
$$F_2\cos\phi = 3.196 \qquad (1)$$

$$+\uparrow\Sigma(F_R)_y = \Sigma F_y; \quad -3 = -3.464 + F_2\sin\phi - 3$$

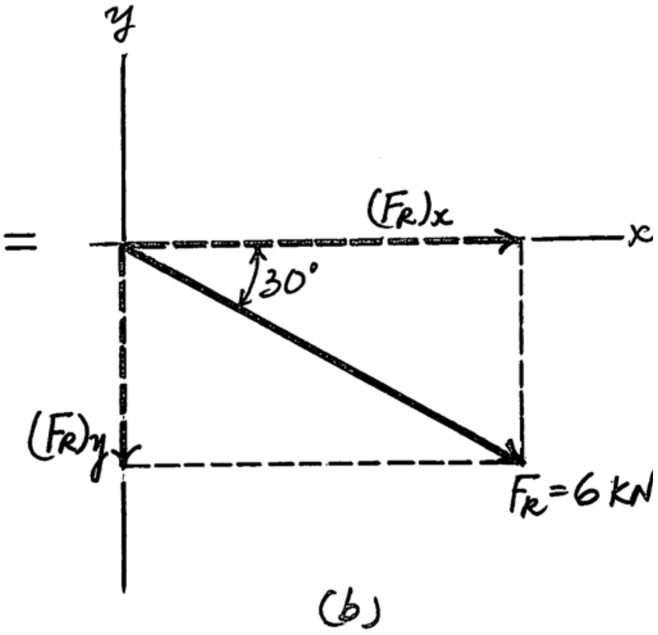
$$F_2\sin\phi = 3.464 \qquad (2)$$

Solving Eqs. (1) and (2), yields

$\phi = 47.3°$ $\qquad F_2 = 4.71$ kN **Ans.**

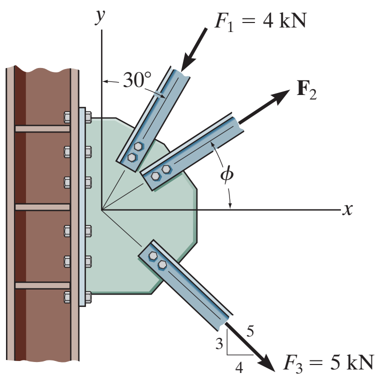


(a)



(b)

2–38. If $\phi = 30^\circ$ and the resultant force acting on the gusset plate is directed along the positive x axis, determine the magnitudes of $\mathbf{F}_2$ and the resultant force.



Rectangular Components: By referring to Fig. *a* , the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, and $\mathbf{F}_R$ can be written as

$$(F_1)_x = 4\sin 30^\circ = 2\text{ kN} \qquad (F_1)_y = 4\cos 30^\circ = 3.464\text{ kN}$$

$$(F_2)_x = F_2\cos 30^\circ = 0.8660F_2 \qquad (F_2)_y = F_2\sin 30^\circ = 0.5F_2$$

$$(F_3)_x = 5\left(\frac{4}{5}\right) = 4\text{ kN} \qquad (F_3)_y = 5\left(\frac{3}{5}\right) = 3\text{kN}$$

$$(F_R)_x = F_R \qquad (F_R)_y = 0$$

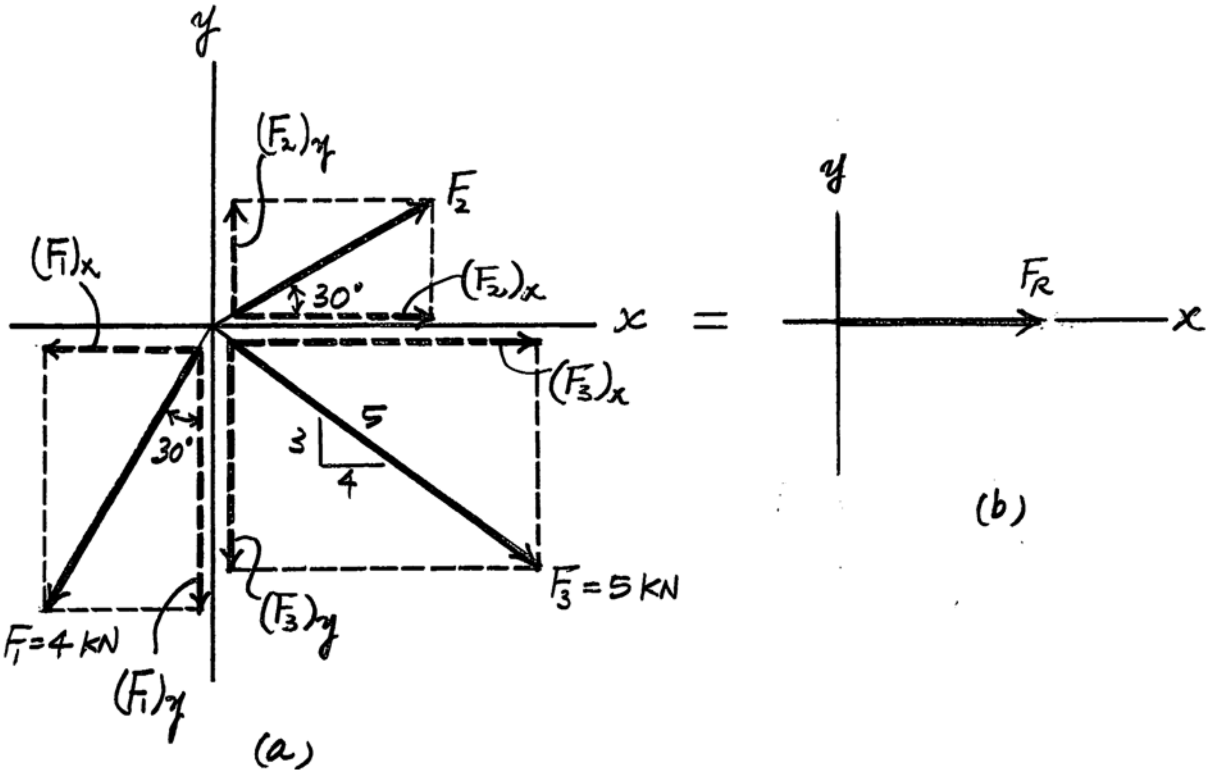
Resultant Force: Summing the force components algebraically along the x and y axes,

$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \quad 0 = -3.464 + 0.5F_2 - 3$$

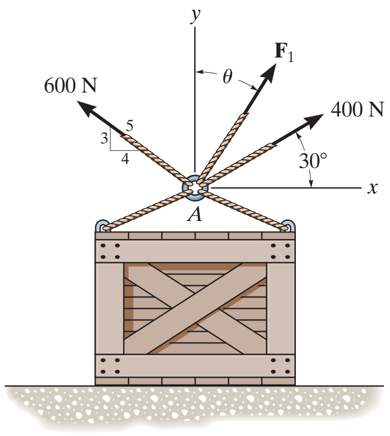
$$F_2 = 12.93\text{ kN} = 12.9\text{ kN} \qquad \textbf{Ans.}$$

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \quad F_R = -2 + 0.8660(12.93) + 4$$

$$= 13.2\text{ kN} \qquad \textbf{Ans.}$$



2–39. Determine the magnitude of $\mathbf{F}_1$ and its direction θ so that the resultant force is directed vertically upward and has a magnitude of 800 N.



Scalar Notation : Suming the force components algebraically, we have

$$\overset{+}{\rightarrow} F_{R_x} = \Sigma F_x ; \qquad F_{R_x} = 0 = F_1 \sin\theta + 400 \cos 30° - 600\left(\frac{4}{5}\right)$$

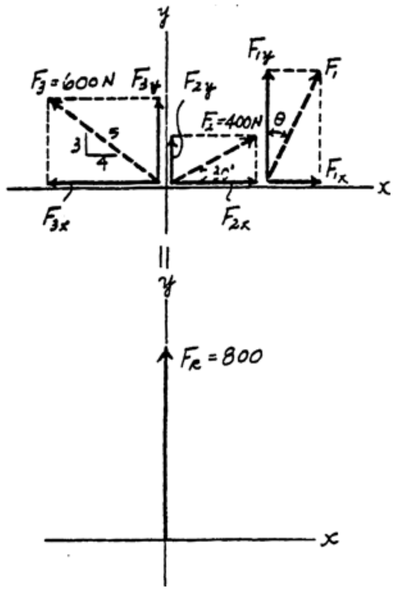
$$F_1 \sin\theta = 133.6 \qquad [1]$$

$$+\uparrow F_{R_y} = \Sigma F_y ; \qquad F_{R_y} = 800 = F_1 \cos\theta + 400 \sin 30° + 600\left(\frac{3}{5}\right)$$

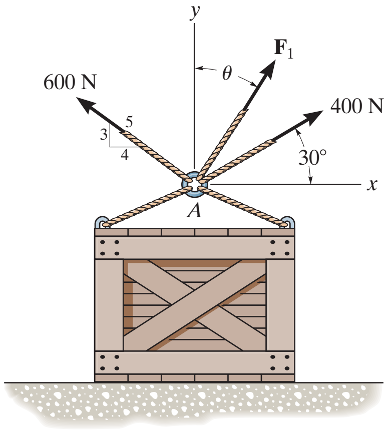
$$F_1 \cos\theta = 240 \qquad [2]$$

Solving Eq. [1] and [2] yields

$$\theta = 29.1° \qquad F_1 = 275\text{ N} \qquad \text{Ans}$$



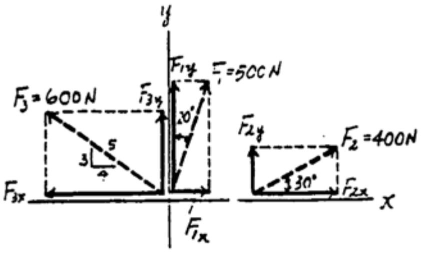
***2–40.** Determine the magnitude and direction measured counterclockwise from the positive x axis of the resultant force of the three forces acting on the ring A. Take $F_1 = 500$ N and $\theta = 20°$.



Scalar Notation : Suming the force components algebraically, we have

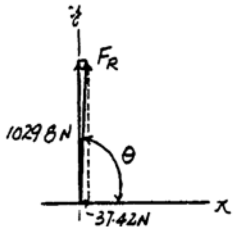
$$\xrightarrow{+} F_{R_x} = \Sigma F_x; \qquad F_{R_x} = 500\sin 20° + 400\cos 30° - 600\left(\frac{4}{5}\right)$$
$$= 37.42 \text{ N} \rightarrow$$

$$+\uparrow F_{R_y} = \Sigma F_y; \qquad F_{R_y} = 500\cos 20° + 400\sin 30° + 600\left(\frac{3}{5}\right)$$
$$= 1029.8 \text{ N} \uparrow$$



The magnitude of the resultant force $\mathbf{F}_R$ is

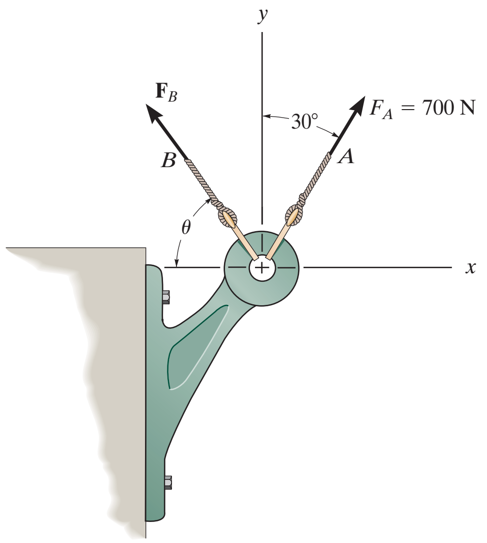
$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{37.42^2 + 1029.8^2} = 1030.5 \text{ N} = 1.03 \text{ kN} \qquad \textbf{Ans}$$



The directional angle θ measured counterclockwise from positive x axis is

$$\theta = \tan^{-1}\frac{F_{R_y}}{F_{R_x}} = \tan^{-1}\left(\frac{1029.8}{37.42}\right) = 87.9° \qquad \textbf{Ans}$$

•2–41. Determine the magnitude and direction θ of $\mathbf{F}_B$ so that the resultant force is directed along the positive y axis and has a magnitude of 1500 N.



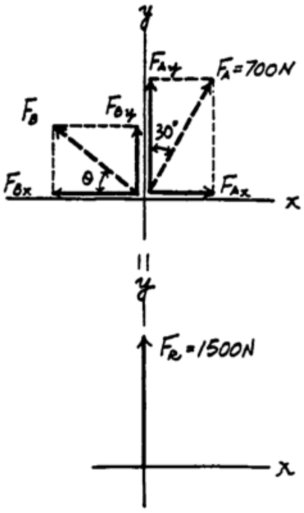
Scalar Notation : **Suming the force components algebraically, we have**

$$\xrightarrow{+} F_{R_x} = \Sigma F_x; \quad 0 = 700 \sin 30° - F_B \cos\theta$$
$$F_B \cos\theta = 350 \qquad [1]$$

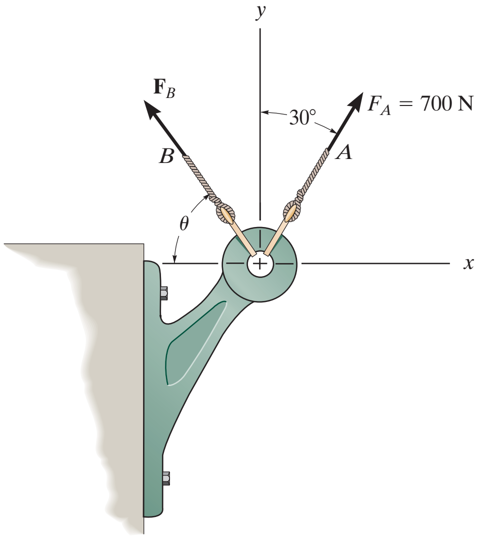
$$+\uparrow F_{R_y} = \Sigma F_y; \quad 1500 = 700 \cos 30° + F_B \sin\theta$$
$$F_B \sin\theta = 893.8 \qquad [2]$$

Solving Eq. [1] and [2] yields

$$\theta = 68.6° \quad F_B = 960 \text{ N} \qquad \textbf{Ans}$$



2–42. Determine the magnitude and angle measured counterclockwise from the positive y axis of the resultant force acting on the bracket if $F_B = 600$ N and $\theta = 20°$.



Scalar Notation : Suming the force components algebraically, we have

$$\xrightarrow{+} F_{R_x} = \Sigma F_x; \quad F_{R_x} = 700 \sin 30° - 600 \cos 20° = \underline{-}213.8 \text{ N} = 213.8 \text{ N} \leftarrow$$

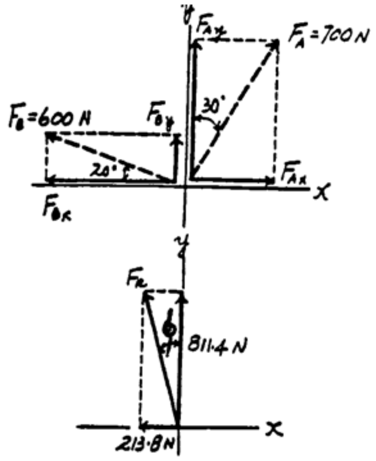
$$+\uparrow F_{R_y} = \Sigma F_y; \quad F_{R_y} = 700 \cos 30° + 600 \sin 20° = 811.4 \text{ N} \uparrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is

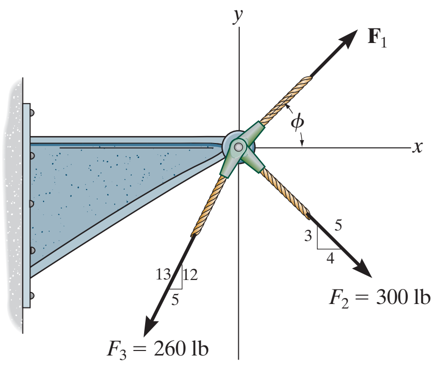
$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{213.8^2 + 811.4^2} = 839 \text{ N} \qquad \textbf{Ans}$$

The directional angle θ measured counterclockwise from positive y axis is

$$\phi = \tan^{-1}\frac{F_{R_x}}{F_{R_y}} = \tan^{-1}\left(\frac{213.8}{811.4}\right) = 14.8° \qquad \textbf{Ans}$$



2–43. If $\phi = 30°$ and $F_1 = 250$ lb, determine the magnitude of the resultant force acting on the bracket and its direction measured clockwise from the positive x axis.



Rectangular Components: By referring to Fig. *a*, the *x* and *y* components of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ can be written as

$$(F_1)_x = 250\cos 30° = 216.51 \text{ lb} \qquad (F_1)_y = 250\sin 30° = 125 \text{ lb}$$

$$(F_2)_x = 300\left(\frac{4}{5}\right) = 240 \text{ lb} \qquad (F_2)_y = 300\left(\frac{3}{5}\right) = 180 \text{ lb}$$

$$(F_3)_x = 260\left(\frac{5}{13}\right) = 100 \text{ lb} \qquad (F_3)_y = 260\left(\frac{12}{13}\right) = 240 \text{ lb}$$

Resultant Force: Summing the force components algebraically along the *x* and *y* axes,

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \quad (F_R)_x = 216.51 + 240 - 100 = 356.51 \text{ lb} \rightarrow$$

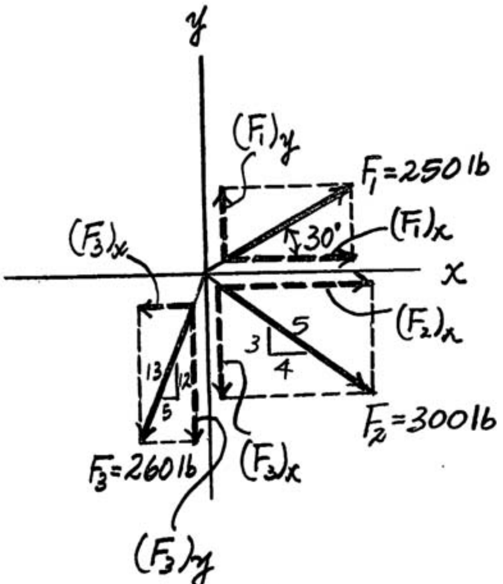
$$+\uparrow\Sigma(F_R)_y = \Sigma F_y; \quad (F_R)_y = 125 - 180 - 240 = -295 \text{ lb} = 295 \text{ lb} \downarrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is

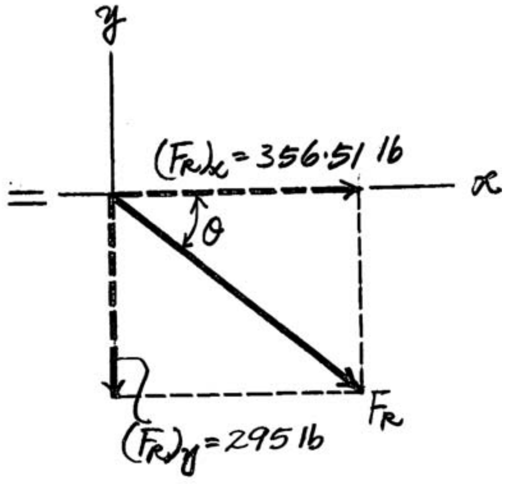
$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2} = \sqrt{356.51^2 + 295^2} = 463 \text{ lb} \qquad \textbf{Ans.}$$

The direction angle θ of $\mathbf{F}_R$, Fig. *b*, measured clockwise from the positive *x* axis, is

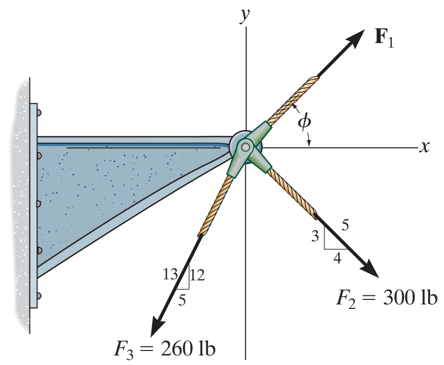
$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{295}{356.51}\right) = 39.6° \qquad \textbf{Ans.}$$



(a)



*__2–44.__ If the magnitude of the resultant force acting on the bracket is 400 lb directed along the positive x axis, determine the magnitude of $\mathbf{F}_1$ and its direction ϕ.



Rectangular Components: By referring to Fig. a, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, and $\mathbf{F}_R$ can be written as

$$(F_1)_x = F_1 \cos\phi \qquad (F_1)_y = F_1 \sin\phi$$

$$(F_2)_x = 300\left(\frac{4}{5}\right) = 240 \text{ lb} \qquad (F_2)_y = 300\left(\frac{3}{5}\right) = 180 \text{ lb}$$

$$(F_3)_x = 260\left(\frac{5}{13}\right) = 100 \text{ lb} \qquad (F_3)_y = 260\left(\frac{12}{13}\right) = 240 \text{ lb}$$

$$(F_R)_x = 400 \text{ lb} \qquad (F_R)_y = 0$$

Resultant Force: Summing the force components algebraically along the x and y axes,

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \quad 400 = F_1 \cos\phi + 240 - 100$$

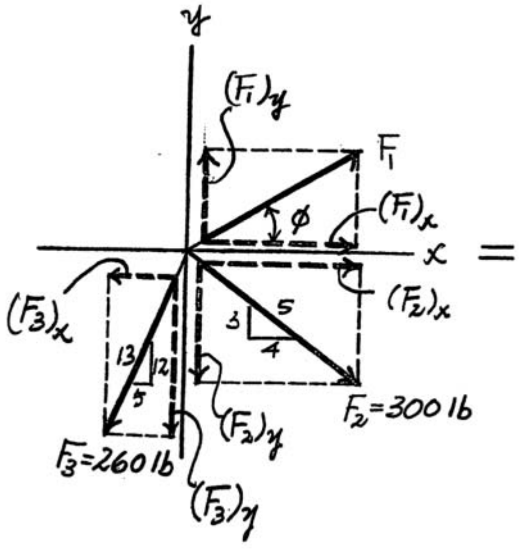
$$F_1 \cos\phi = 260 \qquad (1)$$

$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \quad 0 = F_1 \sin\phi - 180 - 240$$

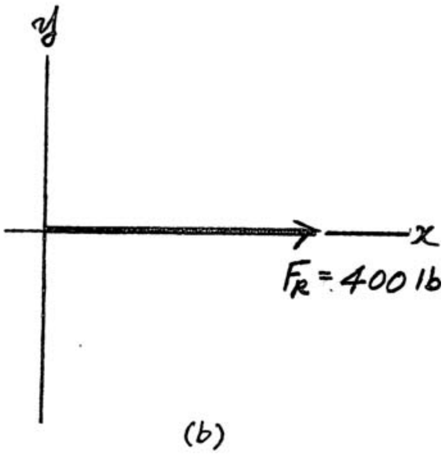
$$F_1 \sin\phi = 420 \qquad (2)$$

Solving Eqs. (1) and (2), yields

$\phi = 58.2°$ $\quad F_1 = 494$ lb **Ans.**

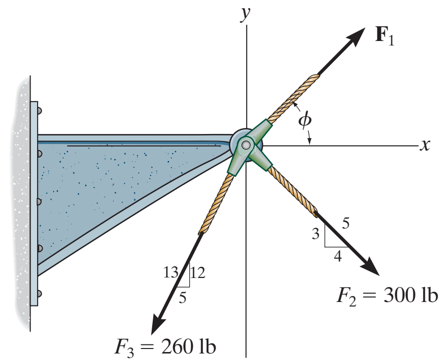


(a)



(b)

•2–45. If the resultant force acting on the bracket is to be directed along the positive x axis and the magnitude of $\mathbf{F}_1$ is required to be a minimum, determine the magnitudes of the resultant force and $\mathbf{F}_1$.



Rectangular Components: By referring to Figs. *a* and *b*, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, and $\mathbf{F}_R$ can be written as

$$(F_1)_x = F_1\cos\phi \qquad (F_1)_y = F_1\sin\phi$$

$$(F_2)_x = 300\left(\frac{4}{5}\right) = 240\text{ lb} \qquad (F_2)_y = 300\left(\frac{3}{5}\right) = 180\text{ lb}$$

$$(F_3)_x = 260\left(\frac{5}{13}\right) = 100\text{ lb} \qquad (F_3)_y = 260\left(\frac{12}{13}\right) = 240\text{ lb}$$

$$(F_R)_x = F_R \qquad (F_R)_y = 0$$

Resultant Force: Summing the force components algebraically along the x and y axes,

$$+\uparrow \Sigma(F_R)_y = \Sigma F_y;\quad 0 = F_1\sin\phi - 180 - 240$$

$$F_1 = \frac{420}{\sin\phi} \qquad (1)$$

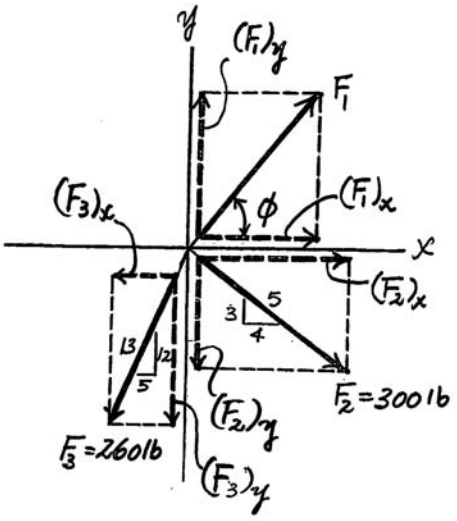
$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x;\quad F_R = F_1\cos\phi + 240 - 100 \qquad (2)$$

By inspecting Eq. (1), we realize that F_1 is minimum when $\sin\phi = 1$ or $\phi = 90°$. Thus,

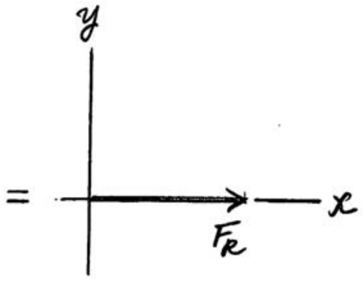
$F_1 = 420$ lb **Ans.**

Substituting these results into Eq. (2), yields

$F_R = 140$ lb **Ans.**

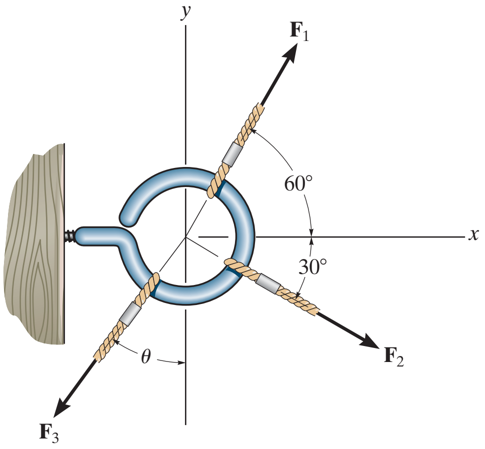


(a)



(b)

2–46. The three concurrent forces acting on the screw eye produce a resultant force $\mathbf{F}_R = 0$. If $F_2 = \frac{2}{3}F_1$ and $\mathbf{F}_1$ is to be 90° from $\mathbf{F}_2$ as shown, determine the required magnitude of $\mathbf{F}_3$ expressed in terms of F_1 and the angle θ.



Cartesian Vector Notation :

$$\mathbf{F}_1 = F_1 \cos 60°\mathbf{i} + F_1 \sin 60°\mathbf{j}$$
$$= 0.50F_1\,\mathbf{i} + 0.8660F_1\,\mathbf{j}$$

$$\mathbf{F}_2 = \frac{2}{3}F_1 \cos 30°\mathbf{i} - \frac{2}{3}F_1 \sin 30°\mathbf{j}$$
$$= 0.5774F_1\,\mathbf{i} - 0.3333F_1\,\mathbf{j}$$

$$\mathbf{F}_3 = -F_3 \sin\theta\,\mathbf{i} - F_3\cos\theta\,\mathbf{j}$$

Resultant Force :

$$\mathbf{F}_R = \mathbf{0} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$
$$0 = (0.50F_1 + 0.5774F_1 - F_3\sin\theta)\,\mathbf{i} + (0.8660F_1 - 0.3333F_1 - F_3\cos\theta)\,\mathbf{j}$$

Equating **i** and **j** components, we have

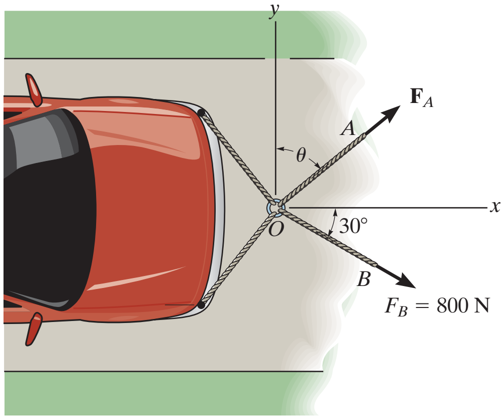
$$0.50F_1 + 0.5774F_1 - F_3\sin\theta = 0 \qquad [1]$$

$$0.8660F_1 - 0.3333F_1 - F_3\cos\theta = 0 \qquad [2]$$

Solving Eq. [1] and [2] yields

$$\theta = 63.7° \qquad F_3 = 1.20F_1 \qquad \textbf{Ans}$$

2–47. Determine the magnitude of $\mathbf{F}_A$ and its direction θ so that the resultant force is directed along the positive x axis and has a magnitude of 1250 N.



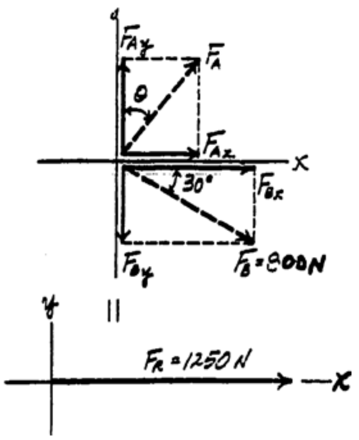
Scalar Notation : Suming the force components algebraically, we have

$$\xrightarrow{+} F_{R_x} = \Sigma F_x; \qquad 1250 = F_A\sin\theta + 800\cos 30°$$
$$F_A\sin\theta = 557.18 \qquad [1]$$

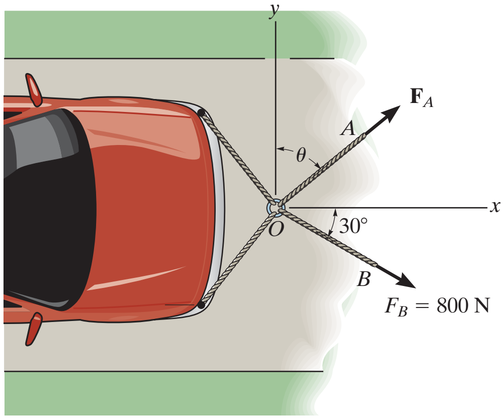
$$+\uparrow F_{R_y} = \Sigma F_y; \qquad 0 = F_A\cos\theta - 800\sin 30°$$
$$F_A\cos\theta = 400 \qquad [2]$$

Solving Eq. [1] and [2] yields

$$\theta = 54.3° \qquad F_A = 686\text{ N} \qquad \textbf{Ans}$$



***2–48.** Determine the magnitude and direction measured counterclockwise from the positive x axis of the resultant force acting on the ring at O if $F_A = 750$ N and $\theta = 45°$.



Scalar Notation : Suming the force components algebraically, we have

$$\overset{+}{\rightarrow} F_{R_x} = \Sigma F_x; \quad F_{R_x} = 750\sin 45° + 800\cos 30° = 1223.15 \text{ N} \rightarrow$$

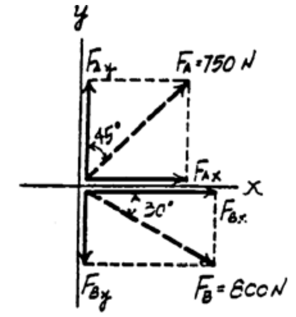
$$+\uparrow F_{R_y} = \Sigma F_y; \quad F_{R_y} = 750\cos 45° - 800\sin 30° = 130.33 \text{ N} \uparrow$$

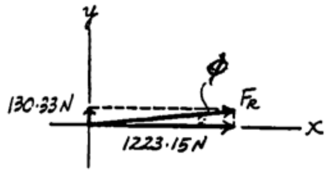
The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{1223.15^2 + 130.33^2} = 1230 \text{ N} = 1.23 \text{ kN} \qquad \textbf{Ans}$$

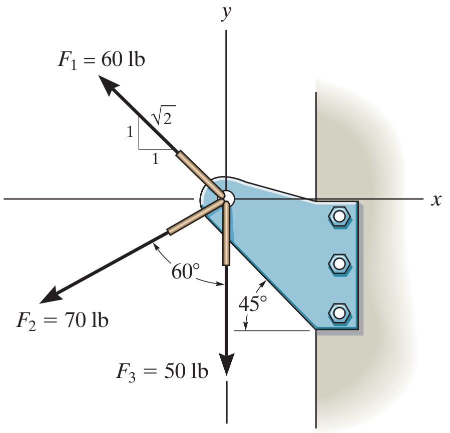
The directional angle θ measured counterclockwise from positive x axis is

$$\phi = \tan^{-1}\frac{F_{R_y}}{F_{R_x}} = \tan^{-1}\left(\frac{130.33}{1223.15}\right) = 6.08° \qquad \textbf{Ans}$$





•2–49. Determine the magnitude of the resultant force and its direction measured counterclockwise from the positive x axis.



$$\mathbf{F}_1 = -60\left(\frac{1}{\sqrt{2}}\right)\mathbf{i} + 60\left(\frac{1}{\sqrt{2}}\right)\mathbf{j} = \{-42.43\mathbf{i} + 42.43\mathbf{j}\} \text{ lb}$$

$$\mathbf{F}_2 = -70\sin 60°\mathbf{i} - 70\cos 60°\mathbf{j} = \{-60.62\,\mathbf{i} - 35\,\mathbf{j}\} \text{ lb}$$

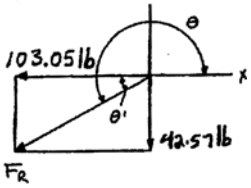
$$\mathbf{F}_3 = \{-50\,\mathbf{j}\} \text{ lb}$$

$$\mathbf{F}_R = \Sigma\mathbf{F} = \{-103.05\,\mathbf{i} - 42.57\,\mathbf{j}\} \text{ lb}$$

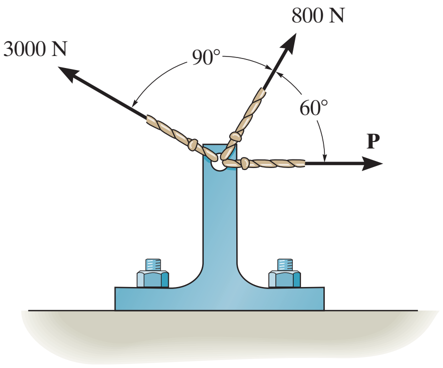
$$F_R = \sqrt{(-103.05)^2 + (-42.57)^2} = 111 \text{ lb} \qquad \textbf{Ans}$$

$$\theta' = \tan^{-1}\left(\frac{42.57}{103.05}\right) = 22.4°$$

$$\theta = 180° + 22.4° = 202° \qquad \textbf{Ans}$$



2–50. The three forces are applied to the bracket. Determine the range of values for the magnitude of force **P** so that the resultant of the three forces does not exceed 2400 N.



$\xrightarrow{+}$ $F_{Rx} = \Sigma F_x$; $\quad F_{Rx} = P + 800\cos 60° - 3000\cos 30° = P - 2198.08$

$+\uparrow F_{Ry} = \Sigma F_y$; $\quad F_{Ry} = 800\sin 60° + 3000\sin 30° = 2192.82$

$$F_R = \sqrt{(P - 2198.08)^2 + (2192.82)^2} \le 2400$$

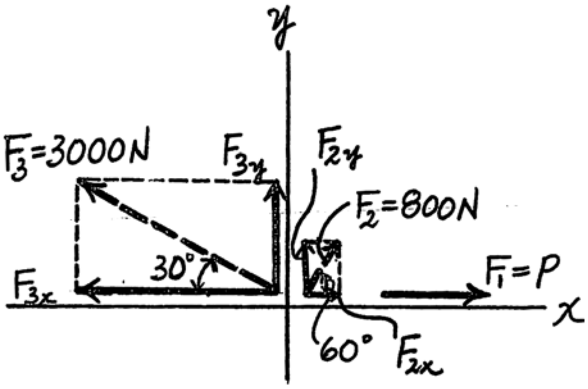
$$(P - 2198.08)^2 + (2192.82)^2 \le (2400)^2$$

$$|(P - 2198.08)| \le 975.47$$

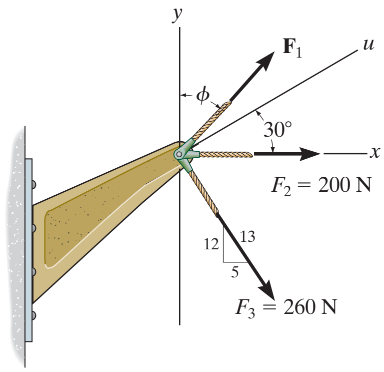
$$-975.47 \le P - 2198.08 \le 975.47$$

$$1222.6\ \text{N} \le P \le 3173.5\ \text{N}$$

$$1.22\ \text{kN} \le P \le 3.17\ \text{kN} \qquad \text{Ans}$$



2–51. If $F_1 = 150$ N and $\phi = 30°$, determine the magnitude of the resultant force acting on the bracket and its direction measured clockwise from the positive x axis.



Rectangular Components: By referring to Fig. *a*, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ can be written as

$$(F_1)_x = 150 \sin 30° = 75\text{ N} \qquad (F_1)_y = 150 \cos 30° = 129.90\text{ N}$$

$$(F_2)_x = 200\text{ N} \qquad (F_2)_y = 0$$

$$(F_3)_x = 260\left(\frac{5}{13}\right) = 100\text{ N} \qquad (F_3)_y = 260\left(\frac{12}{13}\right) = 240\text{ N}$$

Resultant Force: Summing the force components algebraically along the x and y axes,

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \quad (F_R)_x = 75 + 200 + 100 = 375\text{ N} \rightarrow$$

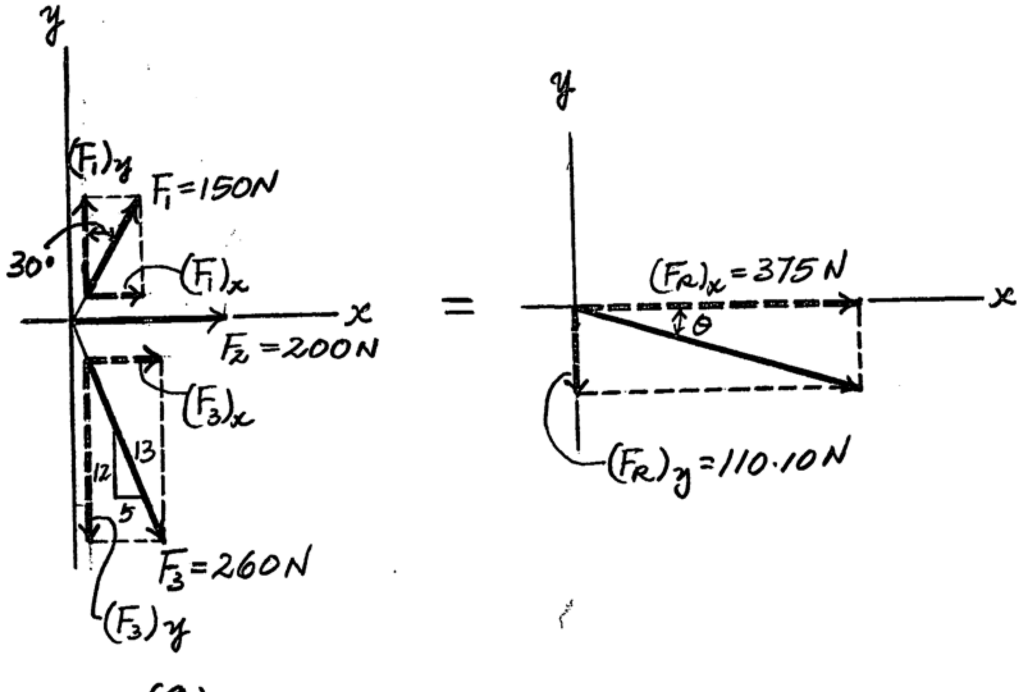
$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \quad (F_R)_y = 129.90 - 240 = -110.10\text{ N} = 110.01\text{ N} \downarrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^{\,2} + (F_R)_y^{\,2}} = \sqrt{375^2 + 110.10^2} = 391\text{N} \qquad \textbf{Ans.}$$

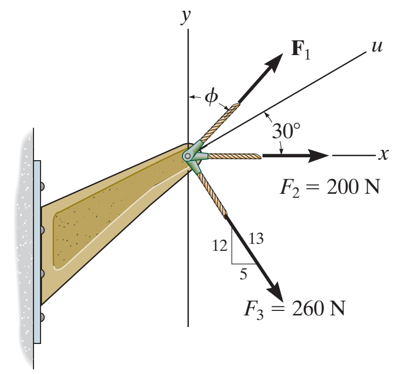
The direction angle θ of $\mathbf{F}_R$, Fig. *b*, measured clockwise from the positive x axis, is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{110.10}{375}\right) = 16.4° \qquad \textbf{Ans.}$$



(a)

***2–52.** If the magnitude of the resultant force acting on the bracket is to be 450 N directed along the positive u axis, determine the magnitude of $\mathbf{F}_1$ and its direction ϕ.



Rectangular Components: By referring to Fig. a, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, and $\mathbf{F}_R$ can be written as

$$(F_1)_x = F_1 \sin\phi \qquad (F_1)_y = F_1 \cos\phi$$

$$(F_2)_x = 200\text{ N} \qquad (F_2)_y = 0$$

$$(F_3)_x = 260\left(\frac{5}{13}\right) = 100\text{ N} \qquad (F_3)_y = 260\left(\frac{12}{13}\right) = 240\text{ lb}$$

$$(F_R)_x = 450\cos 30° = 389.71\text{N} \qquad (F_R)_y = 450\sin 30° = 225\text{ lb}$$

Resultant Force: Summing the force components algebraically along the x and y axes,

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x;\quad 389.71 = F_1\sin\phi + 200 + 100$$

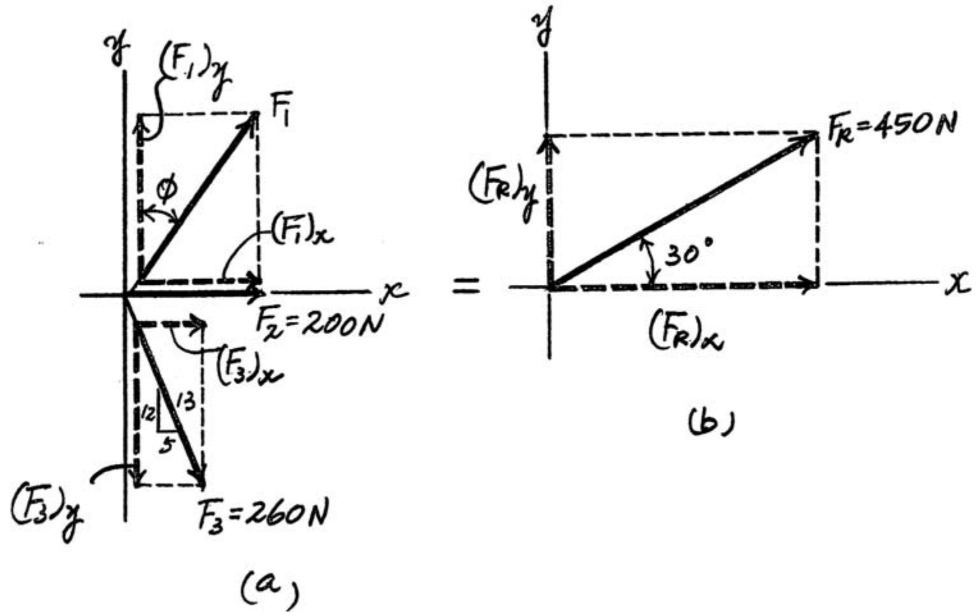
$$F_1 \sin\phi = 89.71 \qquad (1)$$

$$+\uparrow\Sigma(F_R)_y = \Sigma F_y;\quad 225 = F_1\cos\phi - 240$$

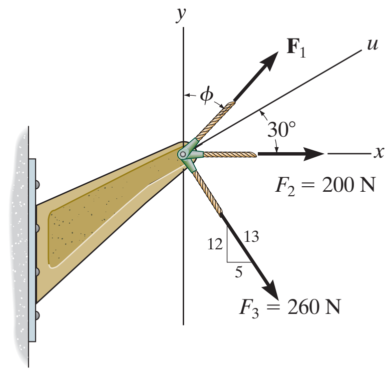
$$F_1\cos\phi = 465 \qquad (2)$$

Solving Eqs. (1) and (2), yields

$\phi = 10.9°$ $\qquad F_1 = 474\text{ N}$ **Ans.**



•2–53. If the resultant force acting on the bracket is required to be a minimum, determine the magnitudes of $\mathbf{F}_1$ and the resultant force. Set $\phi = 30°$.

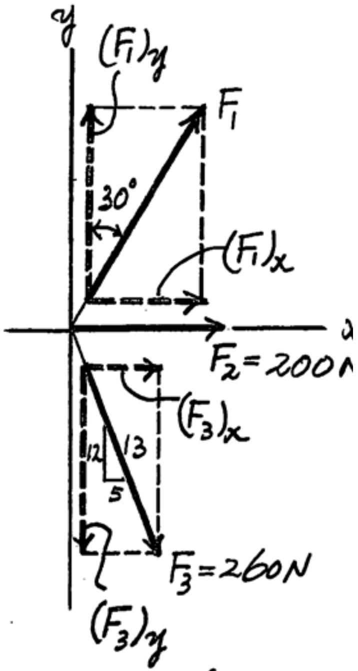


Rectangular Components: By referring to Fig. *a*, the *x* and *y* components of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ can be written as

$$(F_1)_x = F_1 \sin 30° = 0.5F_1 \qquad (F_1)_y = F_1 \cos 30° = 0.8660F_1$$

$$(F_2)_x = 200\text{ N} \qquad (F_2)_y = 0$$

$$(F_3)_x = 260\left(\frac{5}{13}\right) = 100\text{ N} \qquad (F_3)_y = 260\left(\frac{12}{13}\right) = 240\text{ N}$$



(a)

Resultant Force: Summing the force components algebraically along the *x* and *y* axes,

$$\xrightarrow{+}\Sigma(F_R)_x = \Sigma F_x; \quad (F_R)_x = 0.5F_1 + 200 + 100$$
$$= 0.5F_1 + 300$$

$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \quad (F_R)_y = 0.8660F_1 - 240$$

The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2}$$
$$= \sqrt{(0.5F_1 + 300)^2 + (0.8660F_1 - 240)^2}$$
$$= \sqrt{F_1{}^2 - 115.69F_1 + 147600} \qquad (1)$$

Thus,

$$F_R{}^2 = F_1{}^2 - 115.69F_1 + 147\,600 \qquad (2)$$

The first derivative of Eq. (2) is

$$2F_R\frac{dF_R}{dF_1} = 2F_1 - 115.69 \qquad (3)$$

and the second derivative of Eq. (1) is

$$F_R\frac{d^2F_R}{dF_1{}^2} + \frac{dF_R}{dF_1}\frac{dF_R}{dF_1} = 1 \qquad (4)$$

For $\mathbf{F}_R$ to be minimum, $\frac{dF_R}{dF_1} = 0$. Thus, from Eq. (3)

$$2F_R\frac{dF_R}{dF_1} = 2F_1 - 115.69 = 0$$

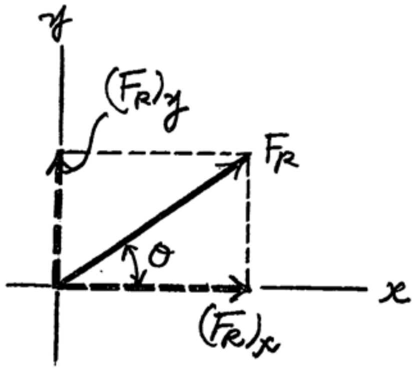
$$F_1 = 57.84\text{ N} = 57.8\text{ N} \qquad \textbf{Ans.}$$

Substituting $F_1 = 57.84$ N and $\frac{dF_R}{dF_1} = 0$ into Eq. (4),

$$\frac{d^2F_R}{dF_1{}^2} = 0.00263 > 0$$

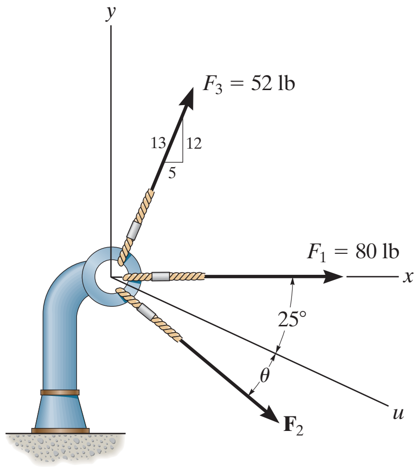
Thus, $F_1 = 57.84$ N produces a minimum $\mathbf{F}_R$. From Eq. (1),

$$F_R = \sqrt{(57.84)^2 - 115.69(57.84) + 147600} = 380\text{ N} \qquad \textbf{Ans.}$$



(b)

2–54. Three forces act on the bracket. Determine the magnitude and direction θ of $\mathbf{F}_2$ so that the resultant force is directed along the positive u axis and has a magnitude of 50 lb.



Scalar Notation : Suming the force components algebraically, we have

$$\xrightarrow{+} F_{R_x} = \Sigma F_x; \quad 50\cos 25° = 80 + 52\left(\frac{5}{13}\right) + F_2\cos(25° + \theta)$$

$$F_2\cos(25° + \theta) = -54.684 \qquad [1]$$

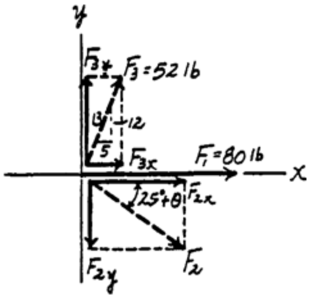
$$+\uparrow F_{R_y} = \Sigma F_y; \quad -50\sin 25° = 52\left(\frac{12}{13}\right) - F_2\sin(25° + \theta)$$

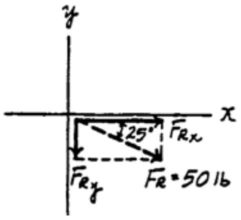
$$F_2\sin(25° + \theta) = 69.131 \qquad [2]$$

Solving Eq.[1] and [2] yields

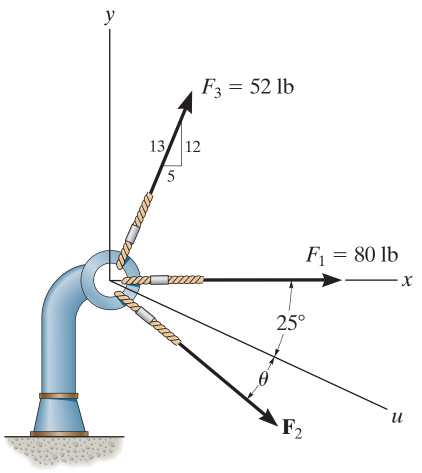
$25° + \theta = 128.35°$ $\quad \theta = 103°$ **Ans**

$F_2 = 88.1$ lb **Ans**





2–55. If $F_2 = 150$ lb and $\theta = 55^\circ$, determine the magnitude and direction measured clockwise from the positive x axis of the resultant force of the three forces acting on the bracket.



Scalar Notation : Suming the force components algebraically, we have

$$\xrightarrow{+} F_{R_x} = \Sigma F_x; \quad F_{R_x} = 80 + 52\left(\frac{5}{13}\right) + 150\cos 80^\circ$$
$$= 126.05 \text{ lb} \rightarrow$$

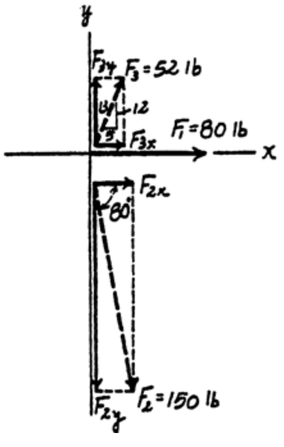
$$+\uparrow F_{R_y} = \Sigma F_y; \quad F_{R_y} = 52\left(\frac{12}{13}\right) - 150\sin 80^\circ$$
$$= -99.72 \text{ lb} = 99.72 \text{ lb} \downarrow$$

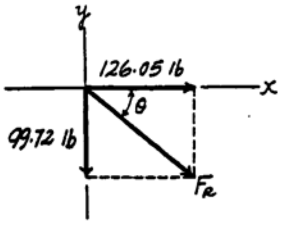
The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{126.05^2 + 99.72^2} = 161 \text{ lb} \qquad \text{Ans}$$

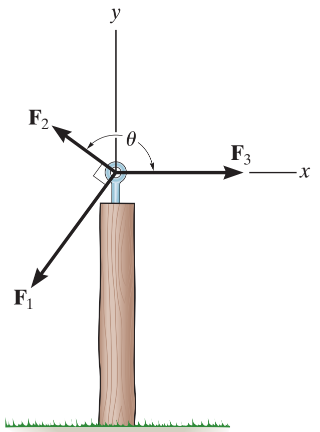
The directional angle θ measured clockwise from positive x axis is

$$\theta = \tan^{-1}\frac{F_{R_y}}{F_{R_x}} = \tan^{-1}\left(\frac{99.72}{126.05}\right) = 38.3^\circ \qquad \text{Ans}$$





***2–56.** The three concurrent forces acting on the post produce a resultant force $\mathbf{F}_R = \mathbf{0}$. If $F_2 = \frac{1}{2} F_1$, and $\mathbf{F}_1$ is to be 90° from $\mathbf{F}_2$ as shown, determine the required magnitude of F_3 expressed in terms of F_1 and the angle θ.



$$\Sigma F_{Rx'} = 0; \qquad F_3 \cos(\theta - 90^\circ) = F_1$$

$$\Sigma F_{Ry'} = 0; \qquad F_3 \sin(\theta - 90^\circ) = F_2$$

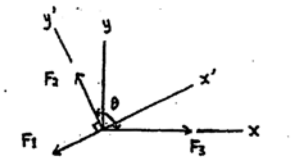
$$\tan(\theta - 90^\circ) = \frac{F_2}{F_1} = \frac{1}{2}$$

$$\theta - 90^\circ = 26.57^\circ$$

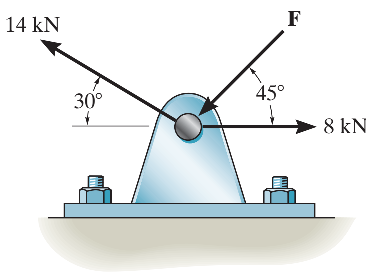
$$\theta = 116.57^\circ = 117^\circ \qquad \text{Ans}$$

$$F_3 = \frac{F_1}{\cos(116.57^\circ - 90^\circ)}$$

$$F_3 = 1.12 F_1 \qquad \text{Ans}$$



•2–57. Determine the magnitude of force **F** so that the resultant force of the three forces is as small as possible. What is the magnitude of this smallest resultant force?



$\xrightarrow{+} F_{Rx} = \Sigma F_x ; \quad F_{Rx} = 8 - F\cos 45° - 14\cos 30°$

$= -4.1244 - F\cos 45°$

$+\uparrow F_{Ry} = \Sigma F_y ; \quad F_{Ry} = -F\sin 45° + 14\sin 30°$

$= 7 - F\sin 45°$

$$F_R^2 = (-4.1244 - F\cos 45°)^2 + (7 - F\sin 45°)^2 \qquad (1)$$

$$2F_R\frac{dF_R}{dF} = 2(-4.1244 - F\cos 45°)(-\cos 45°) + 2(7 - F\sin 45°)(-\sin 45°) = 0$$

$F = 2.03\text{ kN}$ Ans

From Eq. (1); $F_R = 7.87\text{ kN}$ Ans

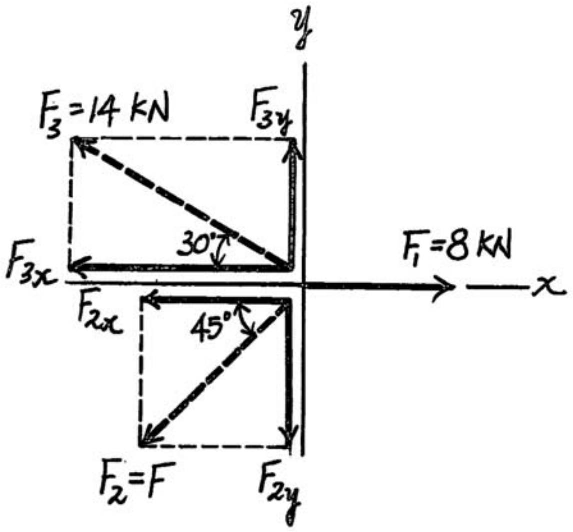
Also, from the figure require

$(F_R)_{x'} = 0 = \Sigma F_{x'} ; \quad F + 14\sin 15° - 8\cos 45° = 0$

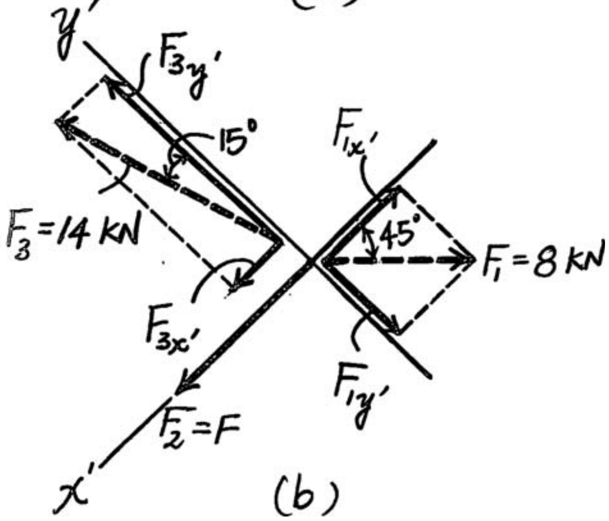
$F = 2.03\text{ kN}$ Ans

$(F_R)_{y'} = \Sigma F_{y'} ; \quad F_R = 14\cos 15° - 8\sin 45°$

$F_R = 7.87\text{ kN}$ Ans

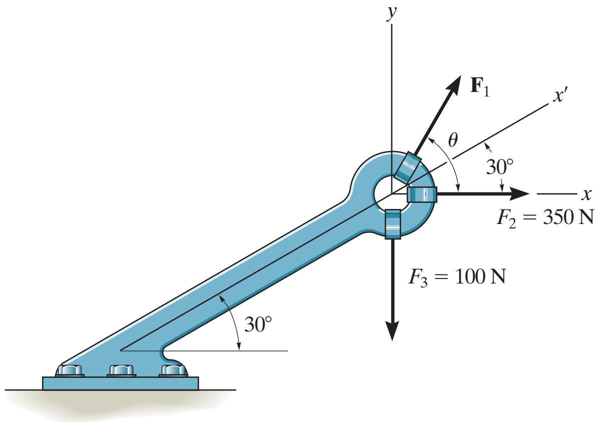


(a)



(b)

2–58. Express each of the three forces acting on the bracket in Cartesian vector form with respect to the x and y axes. Determine the magnitude and direction θ of $\mathbf{F}_1$ so that the resultant force is directed along the positive x' axis and has a magnitude of $F_R = 600$ N.



$$\mathbf{F}_1 = \{F_1 \cos\theta\,\mathbf{i} + F_1 \sin\theta\,\mathbf{j}\}\ \mathrm{N} \qquad \textbf{Ans}$$

$$\mathbf{F}_2 = \{350\mathbf{i}\}\ \mathrm{N} \qquad \textbf{Ans}$$

$$\mathbf{F}_3 = \{-100\mathbf{j}\}\ \mathrm{N} \qquad \textbf{Ans}$$

Require,

$$\mathbf{F}_R = 600 \cos 30^\circ\,\mathbf{i} + 600 \sin 30^\circ\,\mathbf{j}$$

$$\mathbf{F}_R = \{519.6\mathbf{i} + 300\mathbf{j}\}\ \mathrm{N}$$

$$\mathbf{F}_R = \Sigma\mathbf{F}$$

Equating the **i** and **j** components yields:

$$519.6 = F_1 \cos\theta + 350$$

$$F_1 \cos\theta = 169.6$$

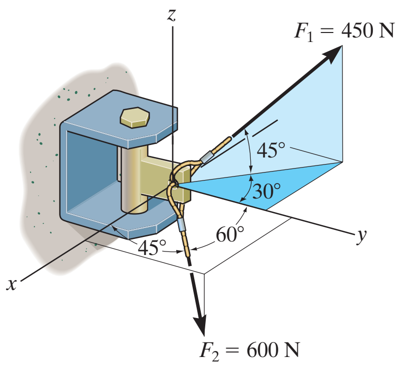
$$300 = F_1 \sin\theta - 100$$

$$F_1 \sin\theta = 400$$

$$\theta = \tan^{-1}\left[\frac{400}{169.6}\right] = 67.0^\circ \qquad \textbf{Ans}$$

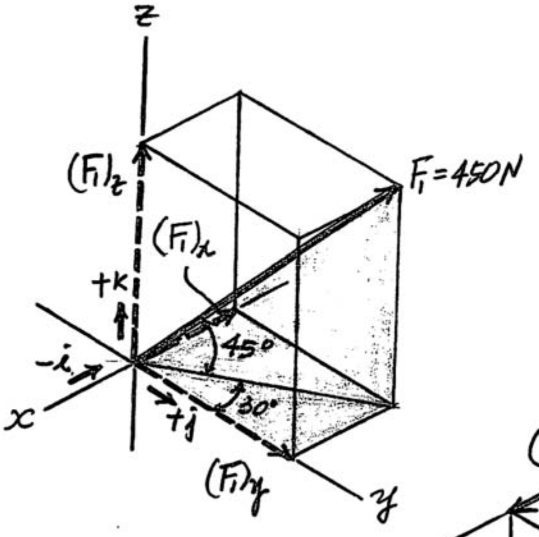
$$F_1 = 434\ \mathrm{N} \qquad \textbf{Ans}$$

2–59. Determine the coordinate angle γ for $\mathbf{F}_2$ and then express each force acting on the bracket as a Cartesian vector.

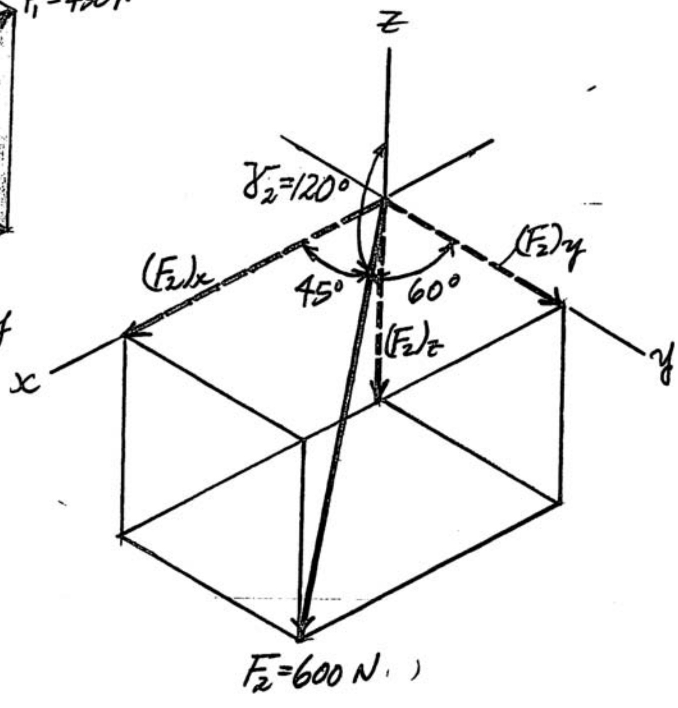


Rectangular Components: Since $\cos^2\alpha_2 + \cos^2\beta_2 + \cos^2\gamma_2 = 1$, then $\cos\gamma_{2z} = \pm\sqrt{1 - \cos^2 45° - \cos^2 60°} = \pm 0.5$. However, it is required that $\gamma_2 > 90°$, thus, $\gamma_2 = \cos^{-1}(-0.5) = 120°$. By resolving $\mathbf{F}_1$ and $\mathbf{F}_2$ into their x, y, and z components, as shown in Figs. a and b, respectively $\mathbf{F}_1$ and $\mathbf{F}_2$ can be expressed in Cartesian vector form as

$$\mathbf{F}_1 = 450\cos 45°\sin 30°(-\mathbf{i}) + 450\cos 45°\cos 30°(+\mathbf{j}) + 450\sin 45°(+\mathbf{k})$$
$$= \{-159\mathbf{i} + 276\mathbf{j} + 318\mathbf{k}\}\text{N} \qquad \textbf{Ans.}$$
$$\mathbf{F}_2 = 600\cos 45°\,\mathbf{i} + 600\cos 60°\,\mathbf{j} + 600\cos 120°\,\mathbf{k}$$
$$= \{424\mathbf{i} + 300\mathbf{j} - 300\mathbf{k}\}\,\text{N} \qquad \textbf{Ans.}$$

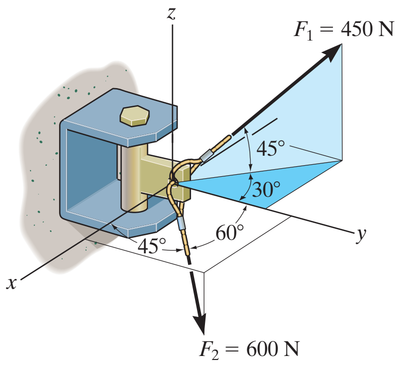


(a)



(b)

***2–60.** Determine the magnitude and coordinate direction angles of the resultant force acting on the bracket.



Rectangular Components: Since $\cos^2\alpha_2 + \cos^2\beta_2 + \cos^2\gamma_2 = 1$, then $\cos\gamma_{2z} = \pm\sqrt{1 - \cos^2 45° - \cos^2 60°} = \pm 0.5$. However, it is required that $\alpha_2 > 90°$, thus, $\gamma_2 = \cos^{-1}(-0.5) = 120°$. By resolving $\mathbf{F}_1$ and $\mathbf{F}_2$ into their x, y, and z components, as shown in Figs. a and b, respectively, $\mathbf{F}_1$ and $\mathbf{F}_2$, can be expressed in Cartesian vector form, as

$$\mathbf{F}_1 = 450\cos 45°\sin 30°(-\mathbf{i}) + 450\cos 45°\cos 30°(+\mathbf{j}) + 450\sin 45°(+\mathbf{k})$$

$$= \{-159.10\mathbf{i} + 275.57\mathbf{j} + 318.20\mathbf{k}\}\text{N} \qquad \textbf{Ans.}$$

$$\mathbf{F}_2 = 600\cos 45°\mathbf{i} + 600\cos 60°\mathbf{j} + 600\cos 120°\mathbf{k}$$

$$= \{424\mathbf{i} + 300\mathbf{j} - 300\mathbf{k}\}\text{ N} \qquad \textbf{Ans.}$$

Resultant Force: By adding $\mathbf{F}_1$ and $\mathbf{F}_2$ vectorally, we obtain $\mathbf{F}_R$.

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$= (-159.10\mathbf{i} + 275.57\mathbf{j} + 318.20\mathbf{k}) + (424.26\mathbf{i} + 300\mathbf{j} - 300\mathbf{k})$$

$$= \{265.16\mathbf{i} + 575.57\mathbf{j} + 18.20\mathbf{k}\}\text{ N}$$

The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^{\,2} + (F_R)_y^{\,2} + (F_R)_z^{\,2}}$$

$$= \sqrt{265.16^2 + 575.57^2 + 18.20^2} = 633.97\text{ N} = 634\text{ N} \qquad \textbf{Ans.}$$

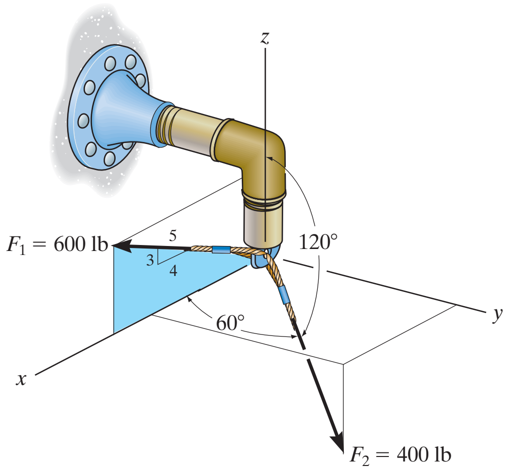
The coordinate direction angles of $\mathbf{F}_R$ are

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{265.16}{633.97}\right) = 65.3° \qquad \textbf{Ans.}$$

$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{575.57}{633.97}\right) = 24.8° \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{18.20}{633.97}\right) = 88.4° \qquad \textbf{Ans.}$$

•2–61. Express each force acting on the pipe assembly in Cartesian vector form.



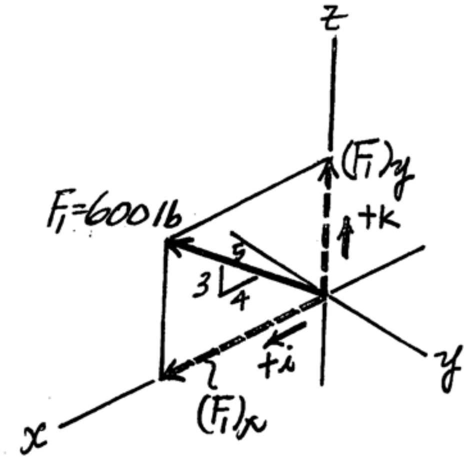
Rectangular Components: Since $\cos^2\alpha_2 + \cos^2\beta_2 + \cos^2\gamma_2 = 1$, then $\cos\beta_2 = \pm\sqrt{1 - \cos^2 60° - \cos^2 120°} = \pm 0.7071$. However, it is required that $\beta_2 > 90°$, thus, $\beta_2 = \cos^{-1}(0.7071) = 45°$. By resolving $\mathbf{F}_1$ and $\mathbf{F}_2$ into their x, y, and z components, as shown in Figs. *a* and *b*, respectively, $\mathbf{F}_1$ and $\mathbf{F}_2$, can be expressed in Cartesian vector form, as

$$\mathbf{F}_1 = 600\left(\frac{4}{5}\right)(+\mathbf{i}) + 0\mathbf{j} + 600\left(\frac{3}{5}\right)(+\mathbf{k})$$

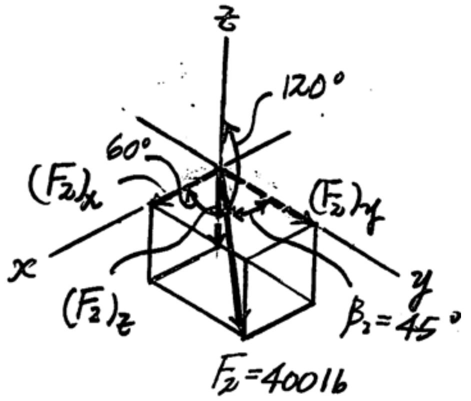
$$= [480\mathbf{i} + 360\mathbf{k}]\text{N} \qquad \textbf{Ans.}$$

$$\mathbf{F}_2 = 400\cos 60°\mathbf{i} + 400\cos 45°\mathbf{j} + 400\cos 120°\mathbf{k}$$

$$= [200\mathbf{i} + 283\mathbf{j} - 200\mathbf{k}]\text{N} \qquad \textbf{Ans.}$$

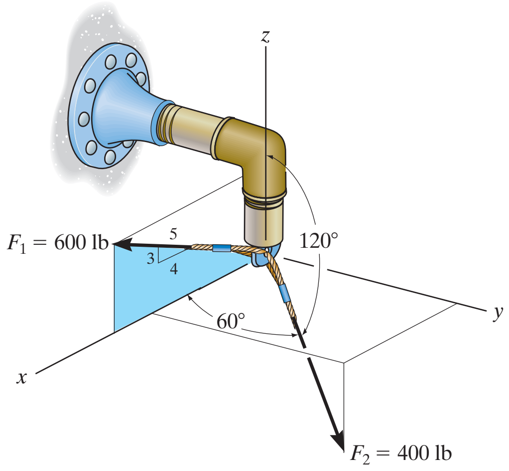


(a)



(b)

2–62. Determine the magnitude and direction of the resultant force acting on the pipe assembly.



Force Vectors: Since $\cos^2\alpha_2+\cos^2\beta_2+\cos^2\gamma_2=1$, then $\cos\gamma_2=\pm\sqrt{1-\cos^2 60°-\cos^2 120°}=\pm 0.7071$. However, it is required that $\gamma_2<90°$, thus, $\gamma_2=\cos^{-1}(0.7071)=45°$. By resolving $\mathbf{F}_1$ and $\mathbf{F}_2$ into their x, y, and z components, as shown in Figs. a and b, respectively, $\mathbf{F}_1$ and $\mathbf{F}_2$, can be expressed in Cartesian vector form, as

$$\mathbf{F}_1=600\left(\frac{4}{5}\right)(+\mathbf{i})+0\mathbf{j}+600\left(\frac{3}{5}\right)(+\mathbf{k})$$
$$=\{480\mathbf{i}+360\mathbf{k}\}\text{ lb}$$
$$\mathbf{F}_2=400\cos 60°\,\mathbf{i}+400\cos 45°\,\mathbf{j}+400\cos 120°\,\mathbf{k}$$
$$=\{200\mathbf{i}+282.84\mathbf{j}-200\mathbf{k}\}\text{ lb}$$

Resultant Force: By adding $\mathbf{F}_1$ and $\mathbf{F}_2$ vectorally, we obtain $\mathbf{F}_R$.

$$\mathbf{F}_R=\mathbf{F}_1+\mathbf{F}_2$$
$$=(480\mathbf{i}+360\mathbf{k})+(200\mathbf{i}+282.84\mathbf{j}-200\mathbf{k})$$
$$=\{680\mathbf{i}+282.84\mathbf{j}+160\mathbf{k}\}\text{ lb}$$

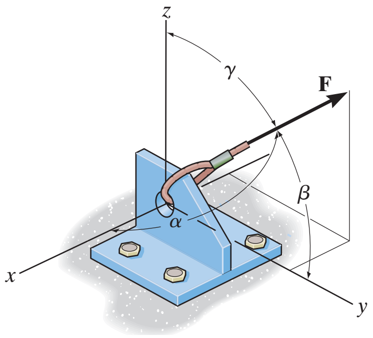
The magnitude of $\mathbf{F}_R$ is

$$F_R=\sqrt{(F_R)_x^{\,2}+(F_R)_y^{\,2}+(F_R)_z^{\,2}}$$
$$=\sqrt{680^2+282.84^2+160^2}=753.66\text{ lb}=754\text{ lb}\qquad\textbf{Ans.}$$

The coordinate direction angles of $\mathbf{F}_R$ are

$$\alpha=\cos^{-1}\left[\frac{(F_R)_x}{F_R}\right]=\cos^{-1}\left(\frac{680}{753.66}\right)=25.5°\qquad\textbf{Ans.}$$
$$\beta=\cos^{-1}\left[\frac{(F_R)_y}{F_R}\right]=\cos^{-1}\left(\frac{282.84}{753.66}\right)=68.0°\qquad\textbf{Ans.}$$
$$\gamma=\cos^{-1}\left[\frac{(F_R)_z}{F_R}\right]=\cos^{-1}\left(\frac{160}{753.66}\right)=77.7°\qquad\textbf{Ans.}$$

2–63. The force **F** acts on the bracket within the octant shown. If $F = 400$ N, $\beta = 60°$, and $\gamma = 45°$, determine the x, y, z components of **F**.



Coordinate Direction Angles: Since β and γ are known, the third angle α can be determined from

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$
$$\cos^2\alpha + \cos^2 60° + \cos^2 45° = 1$$
$$\cos\alpha = \pm 0.5$$

Since **F** is in the octant shown in Fig. *a*, θ_x must be greater than 90°. Thus, $\alpha = \cos^{-1}(-0.5) = 120°$.

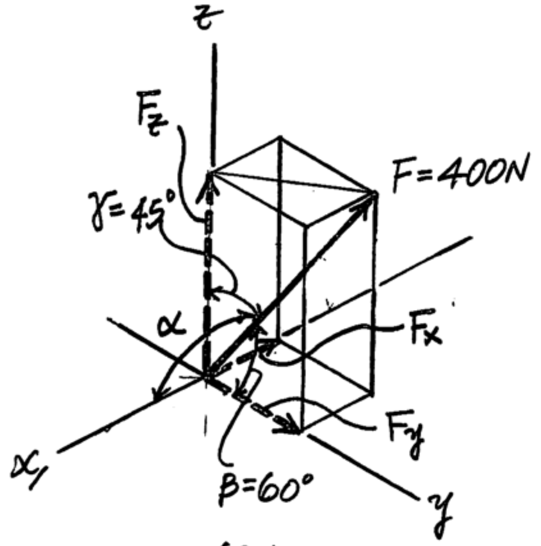
Rectangular Components: By referring to Fig. *a*, the x, y, and z components of **F** can be written as

$F_x = F\cos\alpha = 400\cos 120° = -200\text{ N}$ **Ans.**

$F_y = F\cos\beta = 400\cos 60° = 200\text{ N}$ **Ans.**

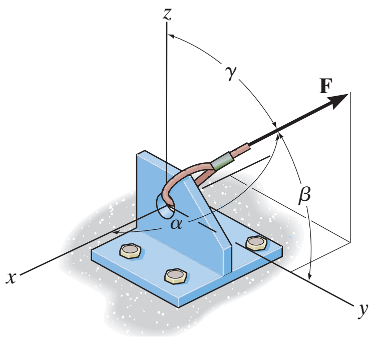
$F_z = F\cos\gamma = 400\cos 45° = 283\text{ N}$ **Ans.**

The negative sign indicates that $\mathbf{F}_x$ is directed towards the negative x axis.



(a)

***2–64.** The force **F** acts on the bracket within the octant shown. If the magnitudes of the x and z components of **F** are $F_x = 300$ N and $F_z = 600$ N, respectively, and $\beta = 60°$, determine the magnitude of **F** and its y component. Also, find the coordinate direction angles α and γ.



Rectangular Components: The magnitude of **F** is given by

$$F = \sqrt{F_x{}^2 + F_y{}^2 + F_z{}^2}$$

$$F = \sqrt{300^2 + F_y{}^2 + 600^2}$$

$$F^2 = F_y{}^2 + 450\,000 \qquad (1)$$

The magnitude of $\mathbf{F}_y$ is given by

$$F_y = F\cos 60° = 0.5F \qquad (2)$$

Solving Eqs. (1) and (2) yields

$$F = 774.60\text{ N} = 775\text{ N} \qquad \textbf{Ans.}$$

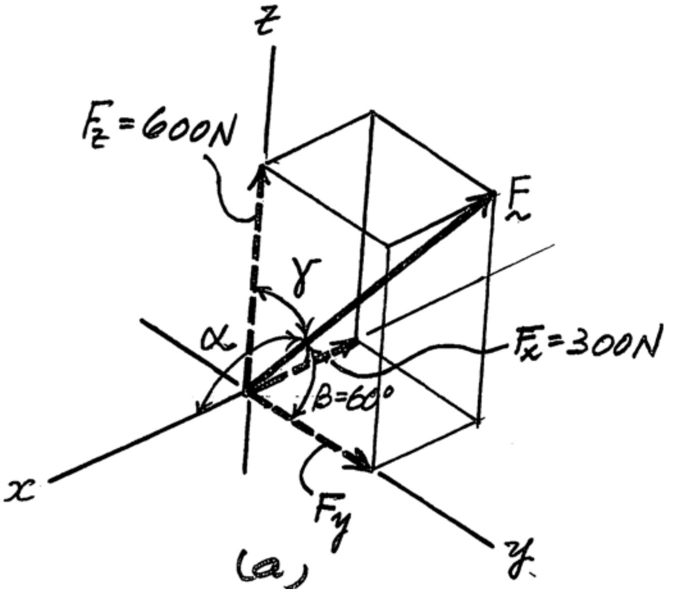
$$F_y = 387\text{ N} \qquad \textbf{Ans.}$$

Coordinate Direction Angles: Since **F** is contained in the octant so that $\mathbf{F}_x$ is directed towards the negative x axis, the coordinate direction angle θ_x is given by

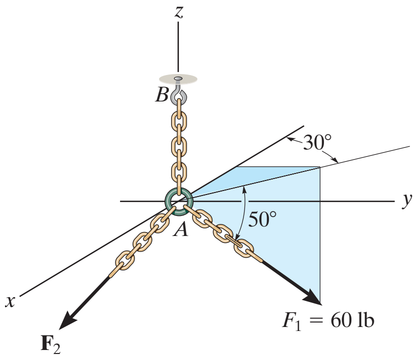
$$\alpha = \cos^{-1}\left(\frac{-F_x}{F}\right) = \cos^{-1}\left(\frac{-300}{774.60}\right) = 113° \qquad \textbf{Ans.}$$

The third coordinate direction angle is

$$\gamma = \cos^{-1}\left(\frac{-F_z}{F}\right) = \cos^{-1}\left(\frac{600}{774.60}\right) = 39.2° \qquad \textbf{Ans.}$$



•2–65. The two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ acting at A have a resultant force of $\mathbf{F}_R = \{-100\mathbf{k}\}$ lb. Determine the magnitude and coordinate direction angles of $\mathbf{F}_2$.



Cartesian Vector Notation :

$$\mathbf{F}_R = \{-100\mathbf{k}\}\text{ lb}$$

$$\mathbf{F}_1 = 60\{-\cos 50^\circ\cos 30^\circ\mathbf{i} + \cos 50^\circ\sin 30^\circ\mathbf{j} - \sin 50^\circ\mathbf{k}\}\text{ lb}$$
$$= \{-33.40\mathbf{i} + 19.28\mathbf{j} - 45.96\mathbf{k}\}\text{ lb}$$

$$\mathbf{F}_2 = \{F_{2_x}\mathbf{i} + F_{2_y}\mathbf{j} + F_{2_z}\mathbf{k}\}\text{ lb}$$

Resultant Force :

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$
$$-100\mathbf{k} = \left\{\left(F_{2_x} - 33.40\right)\mathbf{i} + \left(F_{2_y} + 19.28\right)\mathbf{j} + \left(F_{2_z} - 45.96\right)\mathbf{k}\right\}$$

Equating **i**, **j** and **k** components, we have

$$F_{2_x} - 33.40 = 0 \qquad F_{2_x} = 33.40\text{ lb}$$
$$F_{2_y} + 19.28 = 0 \qquad F_{2_y} = -19.28\text{ lb}$$
$$F_{2_z} - 45.96 = -100 \qquad F_{2_z} = -54.04\text{ lb}$$

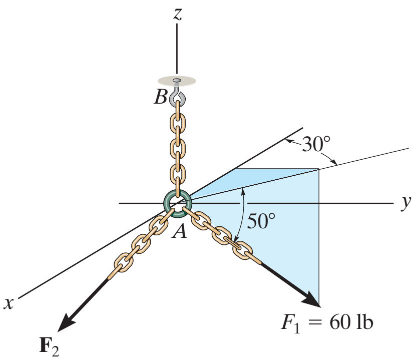
The magnitude of force $\mathbf{F}_2$ is

$$F_2 = \sqrt{F_{2_x}^2 + F_{2_y}^2 + F_{2_z}^2}$$
$$= \sqrt{33.40^2 + (-19.28)^2 + (-54.04)^2}$$
$$= 66.39\text{ lb} = 66.4\text{ lb} \qquad \textbf{Ans}$$

The coordinate direction angles for $\mathbf{F}_2$ are

$$\cos\alpha = \frac{F_{2_x}}{F_2} = \frac{33.40}{66.39} \qquad \alpha = 59.8^\circ \qquad \textbf{Ans}$$
$$\cos\beta = \frac{F_{2_y}}{F_2} = \frac{-19.28}{66.39} \qquad \beta = 107^\circ \qquad \textbf{Ans}$$
$$\cos\gamma = \frac{F_{2_z}}{F_2} = \frac{-54.04}{66.39} \qquad \gamma = 144^\circ \qquad \textbf{Ans}$$

2–66. Determine the coordinate direction angles of the force $\mathbf{F}_1$ and indicate them on the figure.

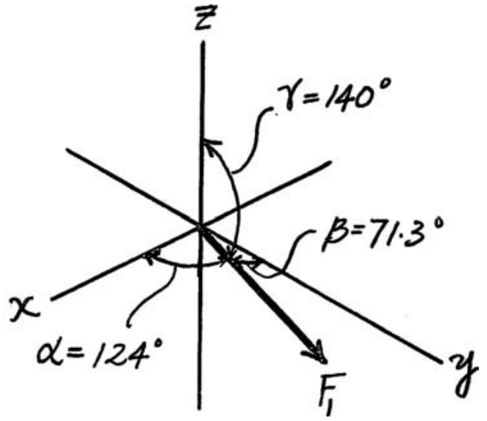


Unit Vector For Foce $\mathbf{F}_1$ **:**

$$\mathbf{u}_{F_1} = -\cos 50^\circ \cos 30^\circ \mathbf{i} + \cos 50^\circ \sin 30^\circ \mathbf{j} - \sin 50^\circ \mathbf{k}$$
$$= -0.5567\mathbf{i} + 0.3214\mathbf{j} - 0.7660\mathbf{k}$$

Coordinate Direction Angles : From the unit vector obtained above, we have

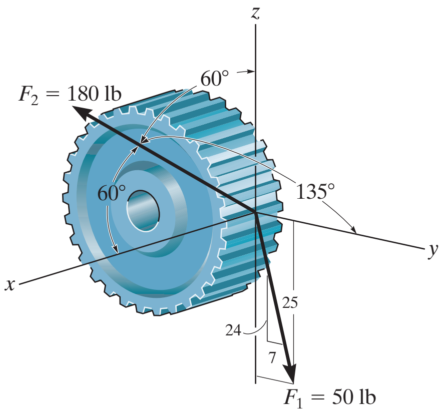
$$\cos\alpha = -0.5567 \qquad \alpha = 124^\circ \qquad \text{Ans}$$
$$\cos\beta = 0.3214 \qquad \beta = 71.3^\circ \qquad \text{Ans}$$
$$\cos\gamma = -0.7660 \qquad \gamma = 140^\circ \qquad \text{Ans}$$



2–67. The spur gear is subjected to the two forces caused by contact with other gears. Express each force as a Cartesian vector.

$$\mathbf{F}_1 = \frac{7}{25}(50)\mathbf{j} - \frac{24}{25}(50)\mathbf{k} = \{14.0\,\mathbf{j} - 48.0\mathbf{k}\}\ \text{lb} \qquad \text{Ans}$$

$$\mathbf{F}_2 = 180\cos 60^\circ\mathbf{i} + 180\cos 135^\circ\mathbf{j} + 180\cos 60^\circ\mathbf{k}$$
$$= \{90\mathbf{i} - 127\mathbf{j} + 90\mathbf{k}\}\ \text{lb} \qquad \text{Ans}$$



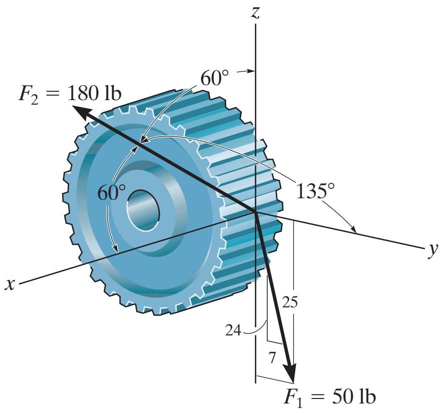
***2–68.** The spur gear is subjected to the two forces caused by contact with other gears. Determine the resultant of the two forces and express the result as a Cartesian vector.

$$F_{Rx} = 180\cos 60^\circ = 90$$

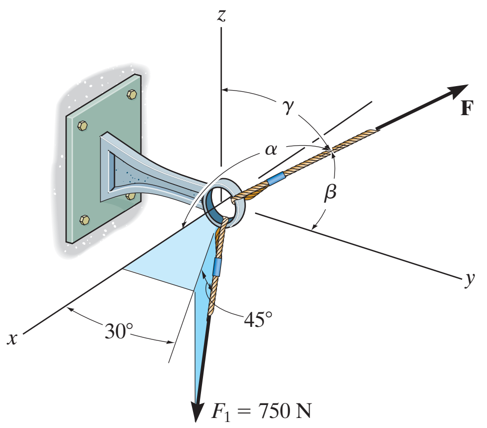
$$F_{Ry} = \frac{7}{25}(50) + 180\cos 135^\circ = -113$$

$$F_{Rz} = -\frac{24}{25}(50) + 180\cos 60^\circ = 42$$

$$\mathbf{F}_R = \{90\mathbf{i} - 113\mathbf{j} + 42\mathbf{k}\}\ \text{lb} \qquad \text{Ans}$$



•2–69. If the resultant force acting on the bracket is $\mathbf{F}_R = \{-300\mathbf{i} + 650\mathbf{j} + 250\mathbf{k}\}$ N, determine the magnitude and coordinate direction angles of **F**.



Force Vectors: By resolving $\mathbf{F}_1$ and $\mathbf{F}_2$ into their x, y, and z components, as shown in Fig. *a*. $\mathbf{F}_1$ and $\mathbf{F}_2$ can be expressed in Cartesian vector form as

$$\mathbf{F}_1 = 750\cos 45°\cos 30°(+\mathbf{i}) + 750\cos 45°\sin 30°(+\mathbf{j}) + 750\sin 45°(-\mathbf{k})$$
$$= [459.28\mathbf{i} + 265.17\mathbf{j} - 530.33\mathbf{k}]\text{N}$$
$$\mathbf{F} = F\cos\alpha\mathbf{i} + F\cos\beta\mathbf{j} + F\cos\gamma\mathbf{k}$$

Resultant Force: By adding $\mathbf{F}_1$ and $\mathbf{F}$ vectorally, we obtain $\mathbf{F}_R$. Thus,

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}$$
$$-300\mathbf{i} + 650\mathbf{j} + 250\mathbf{k} = (459.28\mathbf{i} + 265.17\mathbf{j} - 530.33\mathbf{k}) + (F\cos\theta_x\mathbf{i} + F\cos\theta_y\mathbf{j} + F\cos\theta_z\mathbf{k})$$
$$-300\mathbf{i} + 650\mathbf{j} + 250\mathbf{k} = (459.28 + F\cos\theta_x)\mathbf{i} + (265.17 + F\cos\theta_y)\mathbf{j} + (F\cos\theta_z - 530.33)\mathbf{k}$$

Equating the **i**, **j**, and **k** components,

$$-300 = 459.28 + F\cos\alpha$$
$$F\cos\alpha = -759.28 \qquad (1)$$

$$650 = 265.17 + F\cos\beta$$
$$F\cos\beta = 384.83 \qquad (2)$$

$$250 = F\cos\gamma - 530.33$$
$$F\cos\gamma = 780.33 \qquad (3)$$

Squaring and then adding Eqs. (1), (2), and (3), yields

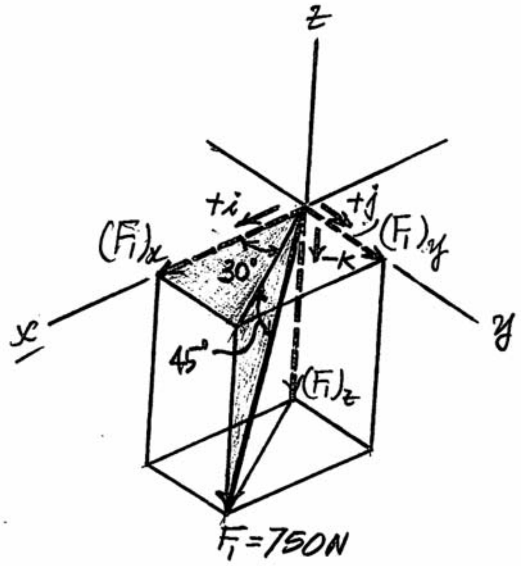
$$F^2(\cos^2\alpha + \cos^2\beta + \cos^2\gamma) = 1\,333\,518.08 \qquad (4)$$

However, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$. Thus, from Eq. (4)

$$F = 1154.78\text{ N} = 1.15\text{ kN} \qquad \textbf{Ans.}$$

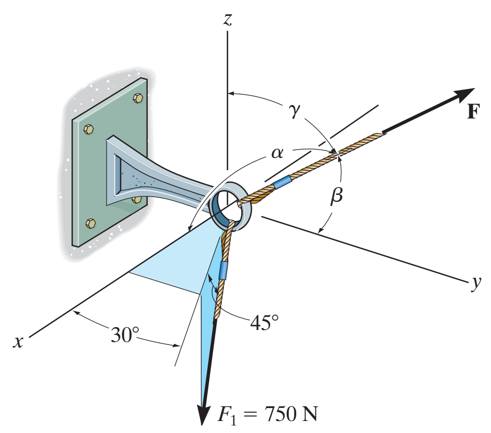
Substituting $F = 1154.78$ N into Eqs. (1), (2), and (3), yields

$$\alpha = 131° \qquad \beta = 70.5° \qquad \gamma = 47.5° \qquad \textbf{Ans.}$$



(a)

2–70. If the resultant force acting on the bracket is to be $\mathbf{F}_R = \{800\mathbf{j}\}$ N, determine the magnitude and coordinate direction angles of **F**.



Force Vectors: By resolving $\mathbf{F}_1$ and $\mathbf{F}$ into their x, y, and z components, as shown in Figs. b and c, respectively, $\mathbf{F}_1$ and $\mathbf{F}$ can be expressed in Cartesian vector form as

$$\mathbf{F}_1 = 750\cos 45°\cos 30°(+\mathbf{i}) + 750\cos 45°\sin 30°(+\mathbf{j}) + 750\sin 45°(-\mathbf{k})$$
$$= [459.28\mathbf{i} + 265.17\mathbf{j} - 530.33\mathbf{k}]\text{N}$$
$$\mathbf{F} = F\cos\alpha\mathbf{i} + F\cos\beta\mathbf{j} + F\cos\gamma\mathbf{k}$$

Resultant Force: By adding $\mathbf{F}_1$ and $\mathbf{F}$ vectorally, Figs. a, b, and c, we obtain $\mathbf{F}_R$. Thus,

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}$$
$$800\mathbf{j} = (459.28\mathbf{i} + 265.17\mathbf{j} - 530.33\mathbf{k}) + (F\cos\alpha\mathbf{i} + F\cos\beta\mathbf{j} + F\cos\gamma\mathbf{k})$$
$$800\mathbf{j} = (459.28 + F\cos\alpha)\mathbf{i} + (265.17 + F\cos\beta)\mathbf{j} + (F\cos\gamma - 530.33)\mathbf{k}$$

Equating the **i**, **j**, and **k** components, we have

$$0 = 459.28 + F_2\cos\alpha$$
$$F\cos\alpha = -459.28 \qquad (1)$$

$$800 = 265.17 + F\cos\beta$$
$$F\cos\beta = 534.8 \qquad (2)$$

$$0 = F\cos\gamma - 530.33$$
$$F\cos\gamma = 530.33 \qquad (3)$$

Squaring and then adding Eqs. (1), (2), and (3), yields

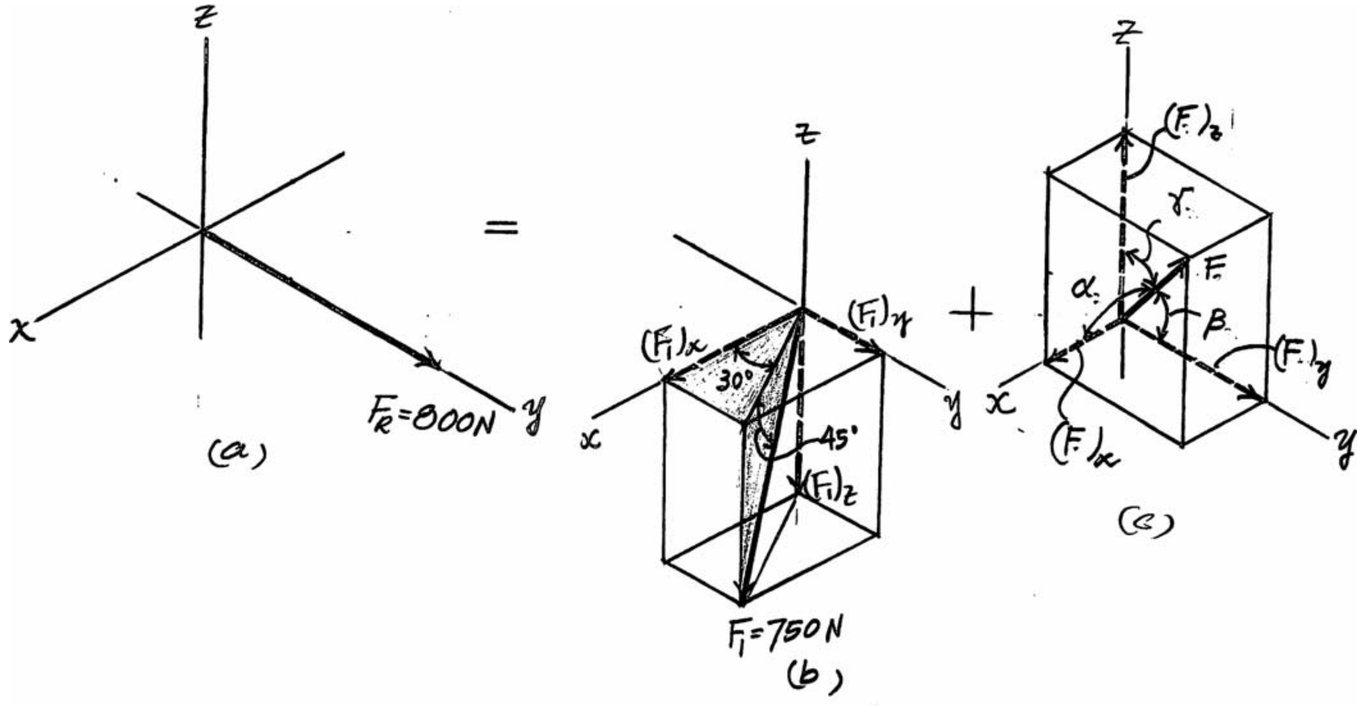
$$F^2(\cos^2\alpha + \cos^2\beta + \cos^2\gamma) = 778\,235.93 \qquad (4)$$

However, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$. Thus, from Eq. (4)

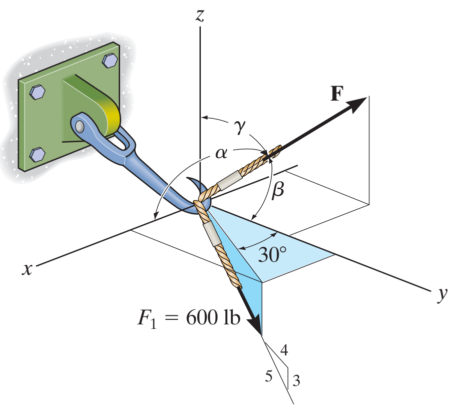
$$F = 882.17\text{ N} = 882\text{ N} \qquad \textbf{Ans.}$$

Substituting $F = 882.17$ N into Eqs. (1), (2), and (3), yields

$$\alpha = 121° \quad \beta = 52.7° \quad \gamma = 53.0° \qquad \textbf{Ans.}$$



2–71. If $\alpha = 120°$, $\beta < 90°$, $\gamma = 60°$, and $F = 400$ lb, determine the magnitude and coordinate direction angles of the resultant force acting on the hook.



Force Vectors: Since $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$, then $\cos\beta = \pm\sqrt{1 - \cos^2 120° - \cos^2 60°} = \pm 0.7071$. However, it is required that $\beta < 90°$, thus, $\beta = \cos^{-1}(0.7071) = 45°$. By resolving $\mathbf{F}_1$ and $\mathbf{F}_2$ into their x, y, and z components, as shown in Figs. *a* and *b*, respectively, $\mathbf{F}_1$ and $\mathbf{F}_2$, can be expressed in Cartesian vector form as

$$\mathbf{F}_1 = 600\left(\frac{4}{5}\right)\sin 30°(+\mathbf{i}) + 600\left(\frac{4}{5}\right)\cos 30°(+\mathbf{j}) + 600\left(\frac{3}{5}\right)(-\mathbf{k})$$
$$= \{240\mathbf{i} + 415.69\mathbf{j} - 360\mathbf{k}\}\text{ lb}$$
$$\mathbf{F} = 400\cos 120°\mathbf{i} + 400\cos 45°\mathbf{j} + 400\cos 60°\mathbf{k}$$
$$= \{-200\mathbf{i} + 282.84\mathbf{j} + 200\mathbf{k}\}\text{ lb}$$

Resultant Force: By adding $\mathbf{F}_1$ and $\mathbf{F}$ vectorally, we obtain $\mathbf{F}_R$.

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}$$
$$= (240\mathbf{i} + 415.69\mathbf{j} - 360\mathbf{k}) + (-200\mathbf{i} + 282.84\mathbf{j} + 200\mathbf{k})$$
$$= \{40\mathbf{i} + 698.53\mathbf{j} - 160\mathbf{k}\}\text{ lb}$$

The magnitude of $\mathbf{F}_R$ is

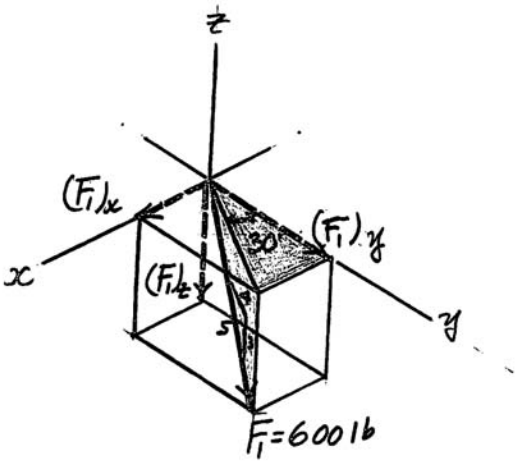
$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2 + (F_R)_z{}^2}$$
$$= \sqrt{(40)^2 + (698.53)^2 + (-160)^2} = 717.74\text{ lb} = 718\text{ lb} \qquad \textbf{Ans.}$$

The coordinate direction angles of $\mathbf{F}_R$ are

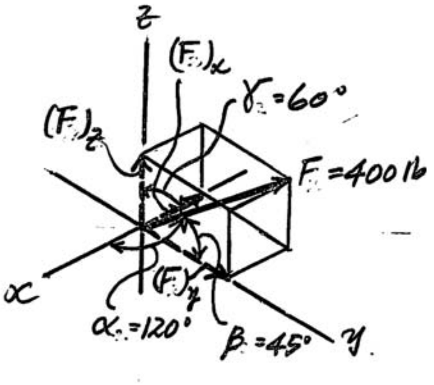
$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{40}{717.74}\right) = 86.8° \qquad \textbf{Ans.}$$

$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{698.53}{717.74}\right) = 13.3° \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{-160}{717.74}\right) = 103° \qquad \textbf{Ans.}$$

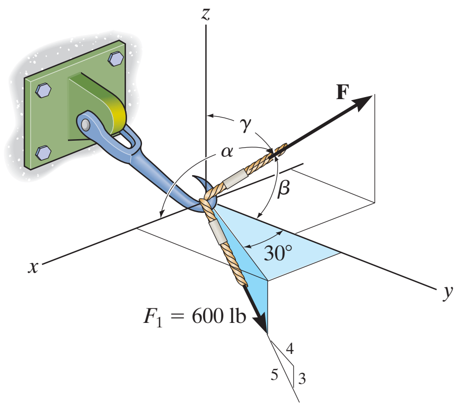


(a)



(b)

***2–72.** If the resultant force acting on the hook is $\mathbf{F}_R = \{-200\mathbf{i} + 800\mathbf{j} + 150\mathbf{k}\}$ lb, determine the magnitude and coordinate direction angles of **F**.



Force Vectors: By resolving $\mathbf{F}_1$ and $\mathbf{F}$ into their x, y, and z components, as shown in Figs. *a* and *b*, respectively, $\mathbf{F}_1$ and $\mathbf{F}_2$ can be expressed in Cartesian vector form as

$$\mathbf{F}_1 = 600\left(\frac{4}{5}\right)\sin 30°(+\mathbf{i}) + 600\left(\frac{4}{5}\right)\cos 30°(+\mathbf{j}) + 600\left(\frac{3}{5}\right)(-\mathbf{k})$$
$$= \{240\mathbf{i} + 415.69\mathbf{j} - 360\mathbf{k}\}\ \text{lb}$$
$$\mathbf{F} = F\cos\alpha\mathbf{i} + F\cos\beta\mathbf{j} + F\cos\gamma\mathbf{k}$$

Resultant Force: By adding $\mathbf{F}_1$ and $\mathbf{F}_2$ vectorally, we obtain $\mathbf{F}_R$. Thus,

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}$$
$$-200\mathbf{i} + 800\mathbf{j} + 150\mathbf{k} = (240\mathbf{i} + 415.69\mathbf{j} - 360\mathbf{k}) + (F\cos\theta_x\mathbf{i} + F\cos\theta_y\mathbf{j} + F\cos\theta_z\mathbf{k})$$
$$-200\mathbf{i} + 800\mathbf{j} + 150\mathbf{k} = (240 + F\cos\alpha)\mathbf{i} + (415.69 + F\cos\beta)\mathbf{j} + (F\cos\gamma - 360)\mathbf{k}$$

Equating the **i**, **j**, and **k** components, we have

$$-200 = 240 + F\cos\theta_x$$
$$F\cos\alpha = -440 \qquad (1)$$

$$800 = 415.69 + F\cos\beta$$
$$F\cos\beta = 384.31 \qquad (2)$$

$$150 = F\cos\gamma - 360$$
$$F\cos\gamma = 510 \qquad (3)$$

Squaring and then adding Eqs. (1), (2), and (3), yields

$$F^2(\cos^2\alpha + \cos^2\beta + \cos^2\gamma) = 601392.49 \qquad (4)$$

However, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$. Thus, from Eq. (4)

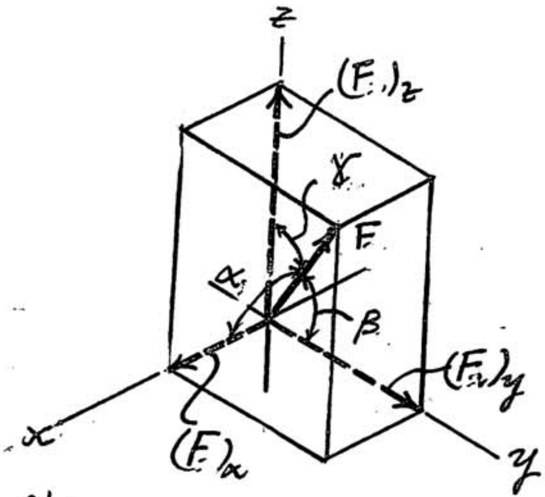
$$F = 775.49\ \text{N} = 775\ \text{N} \qquad \textbf{Ans.}$$

Substituting $F = 775.49$ N into Eqs. (1), (2), and (3), yields

$$\alpha = 125° \qquad \beta = 60.3° \qquad \gamma = 48.9° \qquad \textbf{Ans.}$$

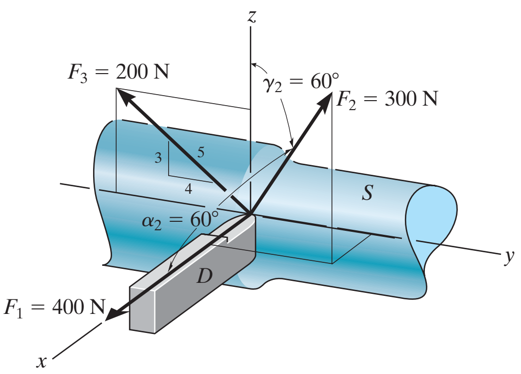
see Prob. 2-71a

(a)



(b)

•2–73. The shaft S exerts three force components on the die D. Find the magnitude and coordinate direction angles of the resultant force. Force $\mathbf{F}_2$ acts within the octant shown.



$\mathbf{F}_1 = 400\,\mathbf{i}$

Since $\cos^2 60° + \cos^2 \beta_2 + \cos^2 60° = 1$

Solving for the positive root, $\beta_2 = 45°$

$\mathbf{F}_2 = 300 \cos 60°\,\mathbf{i} + 300 \cos 45°\,\mathbf{j} + 300 \cos 60°\,\mathbf{k}$

$= 150\,\mathbf{i} + 212.1\,\mathbf{j} + 150\,\mathbf{k}$

$\mathbf{F}_3 = -200\left(\frac{4}{5}\right)\mathbf{j} + 200\left(\frac{3}{5}\right)\mathbf{k}$

$= -160\,\mathbf{j} + 120\,\mathbf{k}$

Then

$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 550\,\mathbf{i} + 52.1\,\mathbf{j} + 270\,\mathbf{k}$

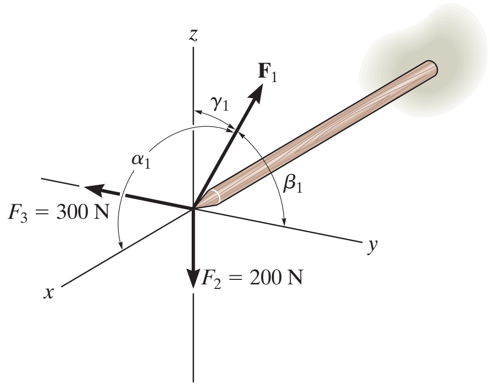
$F_R = \sqrt{(550)^2 + (52.1)^2 + (270)^2} = 614.9 \text{ N} = 615 \text{ N}$ An

$\alpha = \cos^{-1}\left(\frac{550}{614.9}\right) = 26.6°$ **Ans**

$\beta = \cos^{-1}\left(\frac{52.1}{614.9}\right) = 85.1°$ **Ans**

$\gamma = \cos^{-1}\left(\frac{270}{614.9}\right) = 64.0°$ **Ans**

2–74. The mast is subjected to the three forces shown. Determine the coordinate direction angles $\alpha_1, \beta_1, \gamma_1$ of $\mathbf{F}_1$ so that the resultant force acting on the mast is $\mathbf{F}_R = \{350\mathbf{i}\}$ N.



$\mathbf{F}_1 = 500 \cos\alpha_1\,\mathbf{i} + 500\cos\beta_1\,\mathbf{j} + 500 \cos\gamma_1\,\mathbf{k}$

$\mathbf{F}_R = \mathbf{F}_1 + (-300\mathbf{j}) + (-200\mathbf{k})$

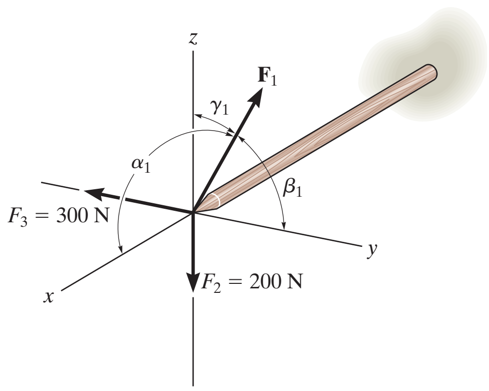
$350\mathbf{i} = 500 \cos\alpha_1\,\mathbf{i} + (500 \cos\beta_1 - 300)\mathbf{j} + (500 \cos\gamma_1 - 200)\mathbf{k}$

$350 = 500 \cos\alpha_1;$ $\alpha_1 = 45.6°$ **Ans**

$0 = 500\cos\beta_1 - 300;$ $\beta_1 = 53.1°$ **Ans**

$0 = 500\cos\gamma_1 - 200;$ $\gamma_1 = 66.4°$ **Ans**

2–75. The mast is subjected to the three forces shown. Determine the coordinate direction angles $\alpha_1, \beta_1, \gamma_1$ of $\mathbf{F}_1$ so that the resultant force acting on the mast is zero.



$\mathbf{F}_1 = \{500\cos\alpha_1\,\mathbf{i} + 500\cos\beta_1\,\mathbf{j} + 500\cos\gamma_1\,\mathbf{k}\}$ N

$\mathbf{F}_2 = \{-200\mathbf{k}\}$ N

$\mathbf{F}_3 = \{-300\mathbf{j}\}$ N

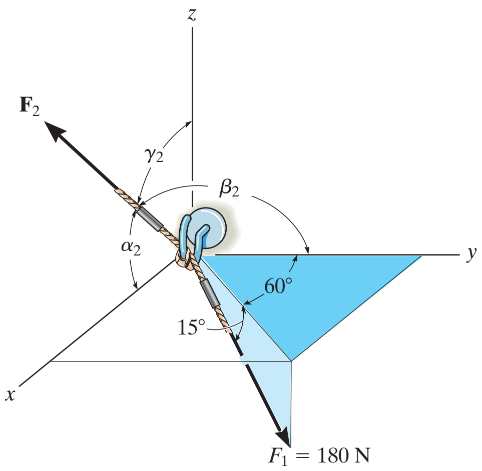
$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = \mathbf{0}$

$500\cos\alpha_1 = 0;$ $\alpha_1 = 90°$ **Ans**

$500\cos\beta_1 = 300;$ $\beta_1 = 53.1°$ **Ans**

$500\cos\gamma_1 = 200;$ $\gamma_1 = 66.4°$ **Ans**

*__2–76.__ Determine the magnitude and coordinate direction angles of $\mathbf{F}_2$ so that the resultant of the two forces acts along the positive x axis and has a magnitude of 500 N.



$\mathbf{F}_1 = (180\cos 15°)\sin 60°\,\mathbf{i} + (180\cos 15°)\cos 60°\,\mathbf{j} - 180\sin 15°\,\mathbf{k}$

$= 150.57\,\mathbf{i} + 86.93\,\mathbf{j} - 46.59\,\mathbf{k}$

$\mathbf{F}_2 = F_2\cos\alpha_2\,\mathbf{i} + F_2\cos\beta_2\,\mathbf{j} + F_2\cos\gamma_2\,\mathbf{k}$

$\mathbf{F}_R = \{500\,\mathbf{i}\}$ N

$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$

i components :

$500 = 150.57 + F_2\cos\alpha_2$

$F_{2x} = F_2\cos\alpha_2 = 349.43$

j components :

$0 = 86.93 + F_2\cos\beta_2$

$F_{2y} = F_2\cos\beta_2 = -86.93$

k components :

$0 = -46.59 + F_2\cos\gamma_2$

$F_{2z} = F_2\cos\gamma_2 = 46.59$

Thus,

$$F_2 = \sqrt{F_{2x}^2 + F_{2y}^2 + F_{2z}^2} = \sqrt{(349.43)^2 + (-86.93)^2 + (46.59)^2}$$

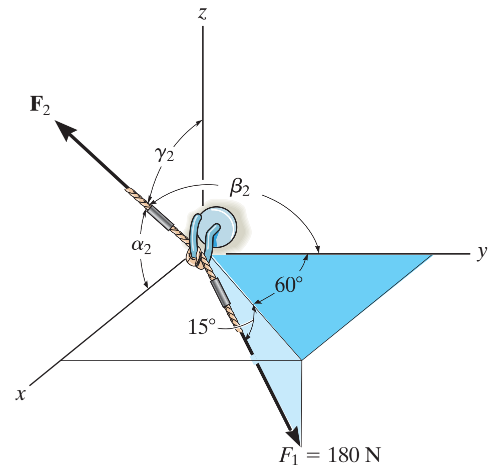
$F_2 = 363$ N **Ans**

$\alpha_2 = 15.8°$ **Ans**

$\beta_2 = 104°$ **Ans**

$\gamma_2 = 82.6°$ **Ans**

•2–77. Determine the magnitude and coordinate direction angles of $\mathbf{F}_2$ so that the resultant of the two forces is zero.



$$\mathbf{F}_1 = (180 \cos 15°) \sin 60° \, \mathbf{i} + (180 \cos 15°) \cos 60° \, \mathbf{j} - 180 \sin 15° \, \mathbf{k}$$

$$= 150.57 \, \mathbf{i} + 86.93 \, \mathbf{j} - 46.59 \, \mathbf{k}$$

$$\mathbf{F}_2 = F_2 \cos \alpha_2 \, \mathbf{i} + F_2 \cos \beta_2 \, \mathbf{j} + F_2 \cos \gamma_2 \, \mathbf{k}$$

$$\mathbf{F}_R = \mathbf{0}$$

i components :

$$0 = 150.57 + F_2 \cos \alpha_2$$

$$F_2 \cos \alpha_2 = -150.57$$

j components :

$$0 = 86.93 + F_2 \cos \beta_2$$

$$F_2 \cos \beta_2 = -86.93$$

k components :

$$0 = -46.59 + F_2 \cos \gamma_2$$

$$F_2 \cos \gamma_2 = 46.59$$

$$F_2 = \sqrt{(-150.57)^2 + (-86.93)^2 + (46.59)^2}$$

Solving,

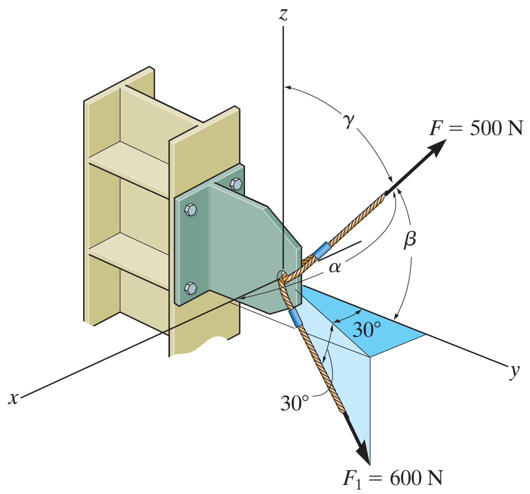
$F_2 = 180$ N **Ans**

$\alpha_2 = 147°$ **Ans**

$\beta_2 = 119°$ **Ans**

$\gamma_2 = 75.0°$ **Ans**

2–78. If the resultant force acting on the bracket is directed along the positive y axis, determine the magnitude of the resultant force and the coordinate direction angles of **F** so that $\beta < 90°$.



Force Vectors: By resolving $\mathbf{F}_1$ and **F** into their x, y, and z components, as shown in Figs. a and b, respectively, $\mathbf{F}_1$ and **F** can be expressed in Cartesian vector form as

$$\mathbf{F}_1 = 600\cos 30°\sin 30°(+\mathbf{i}) + 600\cos 30°\cos 30°(+\mathbf{j}) + 600\sin 30°(-\mathbf{k})$$

$$= \{259.81\mathbf{i} + 450\mathbf{j} - 300\mathbf{k}\}\text{ N}$$

$$\mathbf{F} = 500\cos\alpha\mathbf{i} + 500\cos\beta\mathbf{j} + 500\cos\gamma\mathbf{k}$$

Since the resultant force $\mathbf{F}_R$ is directed towards the positive y axis, then

$$\mathbf{F}_R = F_R\mathbf{j}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}$$

$$F_R\mathbf{j} = (259.81\mathbf{i} + 450\mathbf{j} - 300\mathbf{k}) + (500\cos\theta_x\mathbf{i} + 500\cos\theta_y\mathbf{j} + 500\cos\theta_z\mathbf{k})$$

$$F_R\mathbf{j} = (259.81 + 500\cos\alpha)\mathbf{i} + (450 + 500\cos\beta)\mathbf{j} + (500\cos\gamma - 300)\mathbf{k}$$

Equating the **i**, **j**, and **k** components,

$$0 = 259.81 + 500\cos\alpha$$

$$\alpha = 121.31° = 121° \qquad \textbf{Ans.}$$

$$F_R = 450 + 500\cos\beta \qquad (1)$$

$$0 = 500\cos\gamma - 300$$

$$\gamma = 53.13° = 53.1° \qquad \textbf{Ans.}$$

However, since $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$, $\alpha = 121.31°$, and $\gamma = 53.13°$,

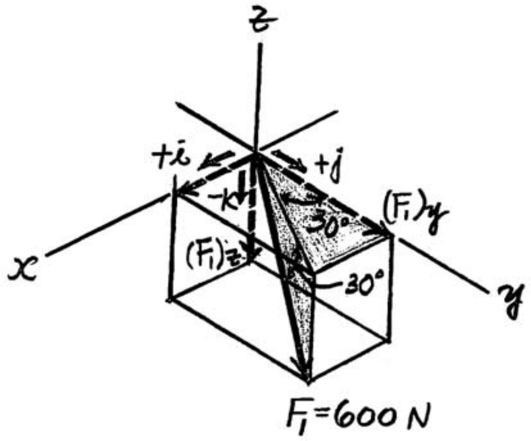
$$\cos\beta = \pm\sqrt{1 - \cos^2 121.31° - \cos^2 53.13°} = \pm 0.6083$$

If we substitute $\cos\beta = 0.6083$ into Eq. (1),

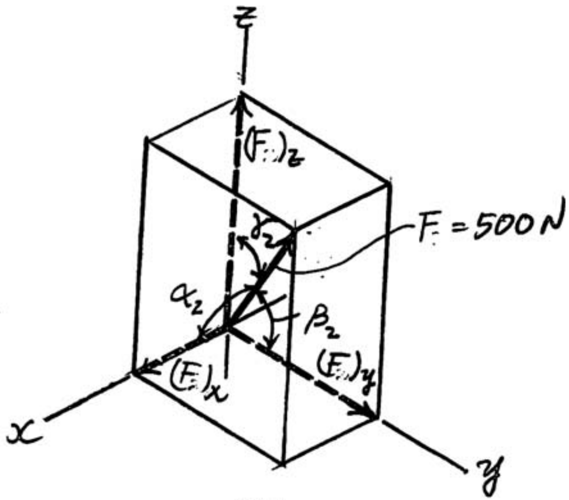
$$F_R = 450 + 500(0.6083) = 754\text{ N} \qquad \textbf{Ans.}$$

and

$$\beta = \cos^{-1}(0.6083) = 52.5° \qquad \textbf{Ans.}$$

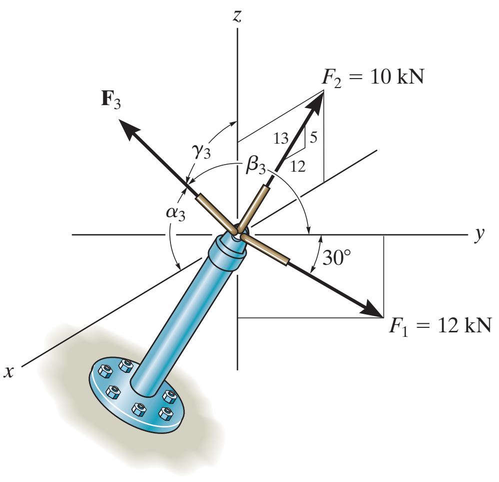


(a)



(b)

2–79. Specify the magnitude of $\mathbf{F}_3$ and its coordinate direction angles $\alpha_3, \beta_3, \gamma_3$ so that the resultant force $\mathbf{F}_R = \{9\mathbf{j}\}$ kN.



$$\mathbf{F}_1 = 12\cos 30^\circ\,\mathbf{j} - 12\sin 30^\circ\,\mathbf{k} = 10.392\,\mathbf{j} - 6\,\mathbf{k}$$

$$\mathbf{F}_2 = -\frac{12}{13}(10)\,\mathbf{i} + \frac{5}{13}(10)\,\mathbf{k} = -9.231\,\mathbf{i} + 3.846\,\mathbf{k}$$

Require

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$

$$9\,\mathbf{j} = 10.392\,\mathbf{j} - 6\,\mathbf{k} - 9.231\,\mathbf{i} + 3.846\,\mathbf{k} + \mathbf{F}_3$$

$$\mathbf{F}_3 = 9.231\,\mathbf{i} - 1.392\,\mathbf{j} + 2.154\,\mathbf{k}$$

Hence,

$$F_3 = \sqrt{(9.231)^2 + (-1.392)^2 + (2.154)^2}$$

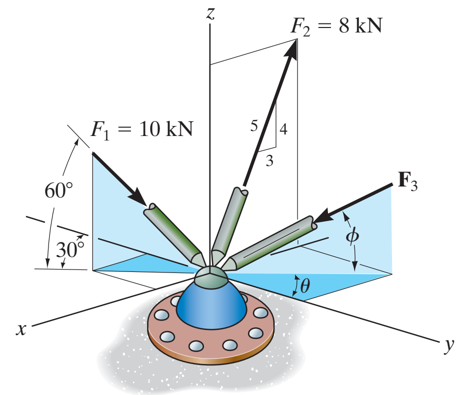
$$F_3 = 9.581\text{ kN} = 9.58\text{ kN} \qquad \textbf{Ans}$$

$$\alpha_3 = \cos^{-1}\left(\frac{9.231}{9.581}\right) = 15.5^\circ \qquad \textbf{Ans}$$

$$\beta_3 = \cos^{-1}\left(\frac{-1.392}{9.581}\right) = 98.4^\circ \qquad \textbf{Ans}$$

$$\gamma_3 = \cos^{-1}\left(\frac{2.154}{9.581}\right) = 77.0^\circ \qquad \textbf{Ans}$$

***2–80.** If $F_3 = 9$ kN, $\theta = 30°$, and $\phi = 45°$, determine the magnitude and coordinate direction angles of the resultant force acting on the ball-and-socket joint.



Force Vectors: By resolving $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ into their x, y, and z components, as shown in Figs. respectively, $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ can be expressed in Cartesian vector form as

$$\mathbf{F}_1 = 10\cos 60°\sin 30°(-\mathbf{i}) + 10\cos 60°\cos 30°(+\mathbf{j}) + 10\sin 60°(-\mathbf{k})$$
$$= \{-2.5\mathbf{i} + 4.330\mathbf{j} - 8.660\mathbf{k}\}\text{ kN}$$

$$\mathbf{F}_2 = 8\left(\frac{3}{5}\right)(-\mathbf{i}) + 0\mathbf{j} + 8\left(\frac{4}{5}\right)(+\mathbf{k})$$
$$= \{-4.8\mathbf{i} + 6.4\mathbf{k}\}\text{ kN}$$

$$\mathbf{F}_3 = 9\cos 45°\sin 30°(+\mathbf{i}) + 9\cos 45°\cos 30°(-\mathbf{j}) + 9\sin 45°(-\mathbf{k})$$
$$= \{3.182\mathbf{i} - 5.511\mathbf{j} - 6.364\mathbf{k}\}\text{ kN}$$

Resultant Force: By adding $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ vectorally, we obtain $\mathbf{F}_R$. Thus,

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$
$$= (-2.5\mathbf{i} + 4.330\mathbf{j} - 8.660\mathbf{k}) + (-4.8\mathbf{i} + 6.4\mathbf{k}) + (3.182\mathbf{i} - 5.511\mathbf{j} - 6.364\mathbf{k})$$
$$= \{-4.118\mathbf{i} - 1.181\mathbf{j} - 8.624\mathbf{k}\}\text{ kN}$$

The magnitude of $\mathbf{F}_R$ is

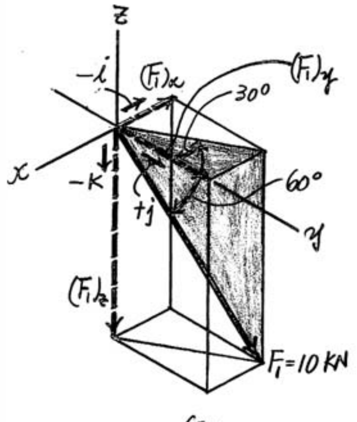
$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2}$$
$$= \sqrt{(-4.118)^2 + (-1.181)^2 + (-8.624)^2} = 9.630\text{ kN} = 9.63\text{ kN} \quad \textbf{Ans.}$$

The coordinate direction angles of $\mathbf{F}_R$ are

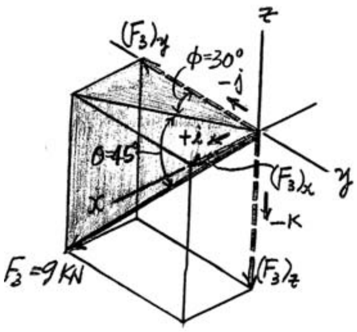
$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{-4.118}{9.630}\right) = 115° \quad \textbf{Ans.}$$

$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{-1.181}{9.630}\right) = 97.0° \quad \textbf{Ans.}$$

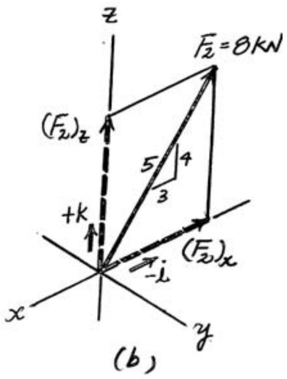
$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{-8.624}{9.630}\right) = 154° \quad \textbf{Ans.}$$



(a)

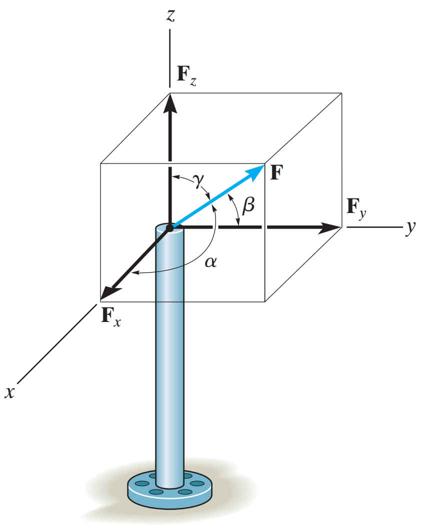


(c)



(b)

•2–81. The pole is subjected to the force **F**, which has components acting along the *x, y, z* axes as shown. If the magnitude of **F** is 3 kN, $\beta = 30°$, and $\gamma = 75°$, determine the magnitudes of its three components.



$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$\cos^2\alpha + \cos^2 30° + \cos^2 75° = 1$$

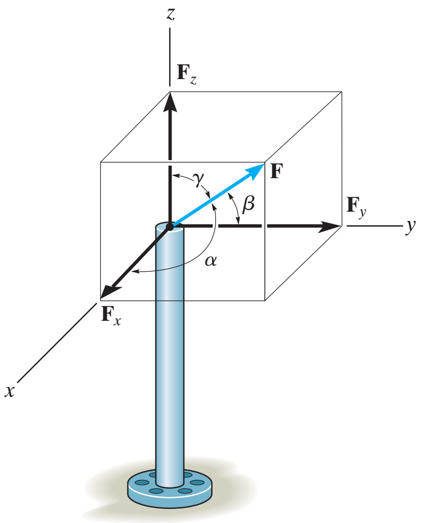
$$\alpha = 64.67°$$

$F_x = 3\cos 64.67° = 1.28 \text{ kN}$ **Ans**

$F_y = 3\cos 30° = 2.60 \text{ kN}$ **Ans**

$F_z = 3\cos 75° = 0.776 \text{ kN}$ **Ans**

2–82. The pole is subjected to the force **F** which has components $F_x = 1.5$ kN and $F_z = 1.25$ kN. If $\beta = 75°$, determine the magnitudes of **F** and $\mathbf{F}_y$.



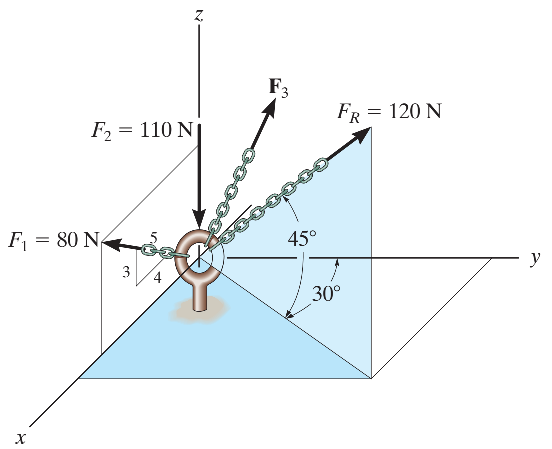
$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$\left(\frac{1.5}{F}\right)^2 + \cos^2 75° + \left(\frac{1.25}{F}\right)^2 = 1$$

$F = 2.02 \text{ kN}$ **Ans**

$F_y = 2.02\cos 75° = 0.523 \text{ kN}$ **Ans**

2–83. Three forces act on the ring. If the resultant force $\mathbf{F}_R$ has a magnitude and direction as shown, determine the magnitude and the coordinate direction angles of force $\mathbf{F}_3$.



Cartesian Vector Notation :

$$\mathbf{F}_R = 120\{\cos 45°\sin 30°\mathbf{i} + \cos 45°\cos 30°\mathbf{j} + \sin 45°\mathbf{k}\}\text{ N}$$
$$= \{42.43\mathbf{i} + 73.48\mathbf{j} + 84.85\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_1 = 80\left\{\frac{4}{5}\mathbf{i} + \frac{3}{5}\mathbf{k}\right\}\text{N} = \{64.0\mathbf{i} + 48.0\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_2 = \{-110\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_3 = \{F_{3_x}\mathbf{i} + F_{3_y}\mathbf{j} + F_{3_z}\mathbf{k}\}\text{ N}$$

Resultant Force :

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$
$$\{42.43\mathbf{i} + 73.48\mathbf{j} + 84.85\mathbf{k}\}$$
$$= \left\{\left(64.0 + F_{3_x}\right)\mathbf{i} + F_{3_y}\mathbf{j} + \left(48.0 - 110 + F_{3_z}\right)\mathbf{k}\right\}$$

Equating **i**, **j** and **k** components, we have

$$64.0 + F_{3_x} = 42.43 \qquad F_{3_x} = -21.57\text{ N}$$
$$F_{3_y} = 73.48\text{ N}$$
$$48.0 - 110 + F_{3_z} = 84.85 \qquad F_{3_z} = 146.85\text{ N}$$

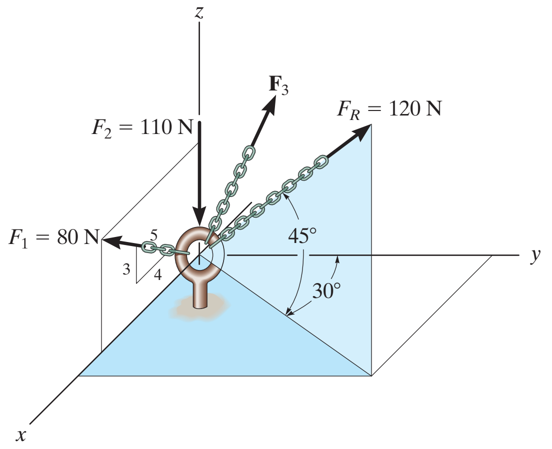
The magnitude of force $\mathbf{F}_3$ is

$$F_3 = \sqrt{F_{3_x}^2 + F_{3_y}^2 + F_{3_z}^2}$$
$$= \sqrt{(-21.57)^2 + 73.48^2 + 146.85^2}$$
$$= 165.62\text{ N} = 166\text{ N} \qquad \textbf{Ans}$$

The coordinate direction angles for $\mathbf{F}_3$ are

$$\cos\alpha = \frac{F_{3_x}}{F_3} = \frac{-21.57}{165.62} \qquad \alpha = 97.5° \qquad \textbf{Ans}$$
$$\cos\beta = \frac{F_{3_y}}{F_3} = \frac{73.48}{165.62} \qquad \beta = 63.7° \qquad \textbf{Ans}$$
$$\cos\gamma = \frac{F_{3_z}}{F_3} = \frac{146.85}{165.62} \qquad \gamma = 27.5° \qquad \textbf{Ans}$$

***2–84.** Determine the coordinate direction angles of $\mathbf{F}_1$ and $\mathbf{F}_R$.



Unit Vector of $\mathbf{F}_1$ and $\mathbf{F}_R$:

$$\mathbf{u}_{F_1} = \frac{4}{5}\mathbf{i} + \frac{3}{5}\mathbf{k} = 0.8\mathbf{i} + 0.6\mathbf{k}$$

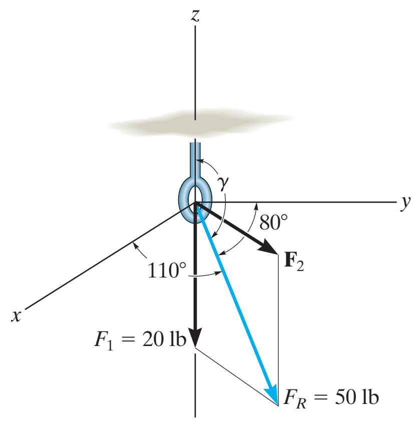
$$\mathbf{u}_R = \cos 45°\sin 30°\mathbf{i} + \cos 45°\cos 30°\mathbf{j} + \sin 45°\mathbf{k}$$
$$= 0.3536\mathbf{i} + 0.6124\mathbf{j} + 0.7071\mathbf{k}$$

Thus, the coordinate direction angles $\mathbf{F}_1$ and $\mathbf{F}_R$ are

$$\cos\alpha_{F_1} = 0.8 \qquad \alpha_{F_1} = 36.9° \qquad \textbf{Ans}$$
$$\cos\beta_{F_1} = 0 \qquad \beta_{F_1} = 90.0° \qquad \textbf{Ans}$$
$$\cos\gamma_{F_1} = 0.6 \qquad \gamma_{F_1} = 53.1° \qquad \textbf{Ans}$$

$$\cos\alpha_R = 0.3536 \qquad \alpha_R = 69.3° \qquad \textbf{Ans}$$
$$\cos\beta_R = 0.6124 \qquad \beta_R = 52.2° \qquad \textbf{Ans}$$
$$\cos\gamma_R = 0.7071 \qquad \gamma_R = 45.0° \qquad \textbf{Ans}$$

•2–85. Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on the bolt. If the resultant force $\mathbf{F}_R$ has a magnitude of 50 lb and coordinate direction angles $\alpha = 110^\circ$ and $\beta = 80^\circ$, as shown, determine the magnitude of $\mathbf{F}_2$ and its coordinate direction angles.



$$(1)^2 = \cos^2 110^\circ + \cos^2 80^\circ + \cos^2 \gamma$$

$$\gamma = 157.44^\circ$$

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$50 \cos 110^\circ = (F_2)_x$$

$$50 \cos 80^\circ = (F_2)_y$$

$$50 \cos 157.44^\circ = (F_2)_z - 20$$

$$(F_2)_x = -17.10$$

$$(F_2)_y = 8.68$$

$$(F_2)_z = -26.17$$

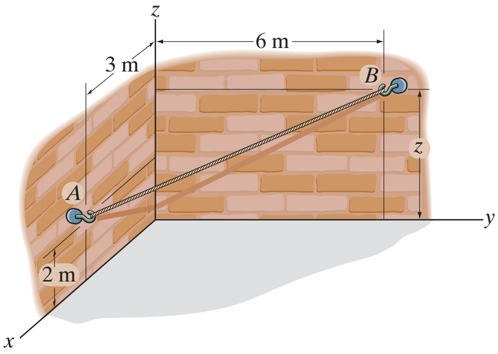
$$F_2 = \sqrt{(-17.10)^2 + (8.68)^2 + (-26.17)^2} = 32.4 \text{ lb} \qquad \textbf{Ans}$$

$$\alpha_2 = \cos^{-1}\left(\frac{-17.10}{32.4}\right) = 122^\circ \qquad \textbf{Ans}$$

$$\beta_2 = \cos^{-1}\left(\frac{8.68}{32.4}\right) = 74.5^\circ \qquad \textbf{Ans}$$

$$\gamma_2 = \cos^{-1}\left(\frac{-26.17}{32.4}\right) = 144^\circ \qquad \textbf{Ans}$$

2–86. Determine the position vector **r** directed from point *A* to point *B* and the length of cord *AB*. Take $z = 4$ m.



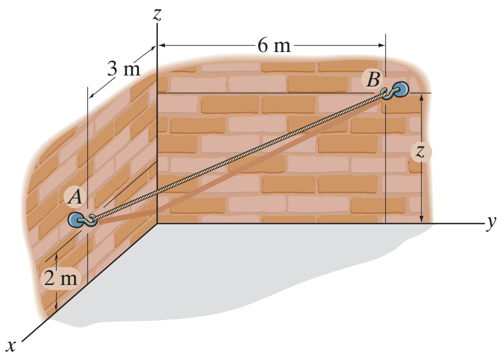
Position Vector: The coordinates for points *A* and *B* are *A*(3, 0, 2) m and *B*(0, 6, 4) m, respectively. Thus,

$$\mathbf{r}_{AB} = (0-3)\mathbf{i} + (6-0)\mathbf{j} + (4-2)\mathbf{k}$$
$$= \{-3\mathbf{i} + 6\mathbf{j} + 2\mathbf{k}\} \text{ m} \qquad \textbf{Ans.}$$

The length of cord *AB* is

$$r_{AB} = \sqrt{(-3)^2 + 6^2 + 2^2} = 7 \text{ m} \qquad \textbf{Ans.}$$

2–87. If the cord AB is 7.5 m long, determine the coordinate position $+z$ of point B



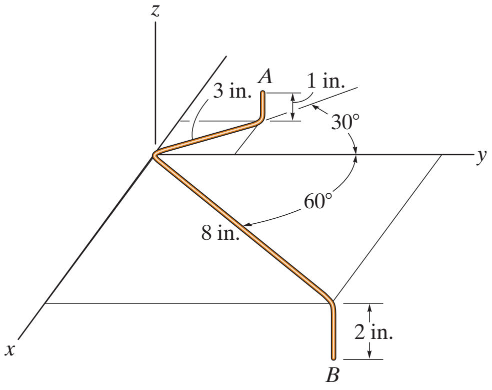
Position Vector: The coordinates for points A and B are $A(3, 0, 2)$ m and $B(0, 6, z)$ m, respectively. Thus,

$$\mathbf{r}_{AB} = (0-3)\mathbf{i}+(6-0)\mathbf{j}+(z-2)\mathbf{k}$$
$$=\{-3\mathbf{i}+6\mathbf{j}+(z-2)\mathbf{k}\}\text{ m}$$

Since the length of cord is equal to the magnitude of $\mathbf{r}_{AB}$, then

$$r_{AB} = 7.5 = \sqrt{(-3)^2+6^2+(z-2)^2}$$
$$56.25 = 45+(z-2)^2$$
$$z-2 = \pm 3.354$$
$$z = 5.35\text{ m} \qquad \textbf{Ans.}$$

***2–88.** Determine the distance between the end points A and B on the wire by first formulating a position vector from A to B and then determining its magnitude.

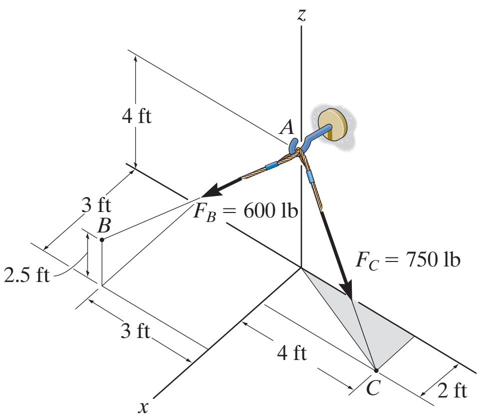


$$\mathbf{r}_{AB} = (8\sin 60° - (-3\sin 30°))\,\mathbf{i} + (8\cos 60° - 3\cos 30°)\,\mathbf{j} + (-2 - 1)\,\mathbf{k}$$

$$\mathbf{r}_{AB} = \{8.428\,\mathbf{i} + 1.402\,\mathbf{j} - 3\,\mathbf{k}\}\text{ in.}$$

$$r_{AB} = \sqrt{(8.428)^2 + (1.402)^2 + (-3)^2} = 9.06\text{ in.} \qquad \textbf{Ans}$$

•2–89. Determine the magnitude and coordinate direction angles of the resultant force acting at A.



Unit Vectors: The coordinate points A, B, and C are shown in Fig. a. Thus,

$$\mathbf{u}_B=\frac{\mathbf{r}_B}{r_B}=\frac{(3-0)\mathbf{i}+(-3-0)\mathbf{j}+(2.5-4)\mathbf{k}}{\sqrt{(3-0)^2+(-3-0)^2+(2.5-4)^2}}$$

$$=\frac{2}{3}\mathbf{i}-\frac{2}{3}\mathbf{j}-\frac{1}{3}\mathbf{k}$$

$$\mathbf{u}_C=\frac{\mathbf{r}_C}{r_C}=\frac{(2-0)\mathbf{i}+(4-0)\mathbf{j}+(0-4)\mathbf{k}}{\sqrt{(2-0)^2+(4-0)^2+(0-4)^2}}$$

$$=\frac{1}{3}\mathbf{i}+\frac{2}{3}\mathbf{j}-\frac{2}{3}\mathbf{k}$$

Force Vectors: Multiplying the magnitude of the force with its unit vector, we have

$$\mathbf{F}_B=F_B\mathbf{u}_B=600\left(\frac{2}{3}\mathbf{i}-\frac{2}{3}\mathbf{j}-\frac{1}{3}\mathbf{k}\right)=\{400\mathbf{i}-400\mathbf{j}-200\mathbf{k}\}\text{ lb} \qquad \textbf{Ans.}$$

$$\mathbf{F}_C=F_C\mathbf{u}_C=750\left(\frac{1}{3}\mathbf{i}+\frac{2}{3}\mathbf{j}-\frac{2}{3}\mathbf{k}\right)=\{250\mathbf{i}+500\mathbf{j}-500\mathbf{k}\}\text{ lb} \qquad \textbf{Ans.}$$

$$\mathbf{F}_R=\mathbf{F}_B+\mathbf{F}_C=400\mathbf{i}-400\mathbf{j}-200\mathbf{k}+250\mathbf{i}+500\mathbf{j}-500\mathbf{k}$$

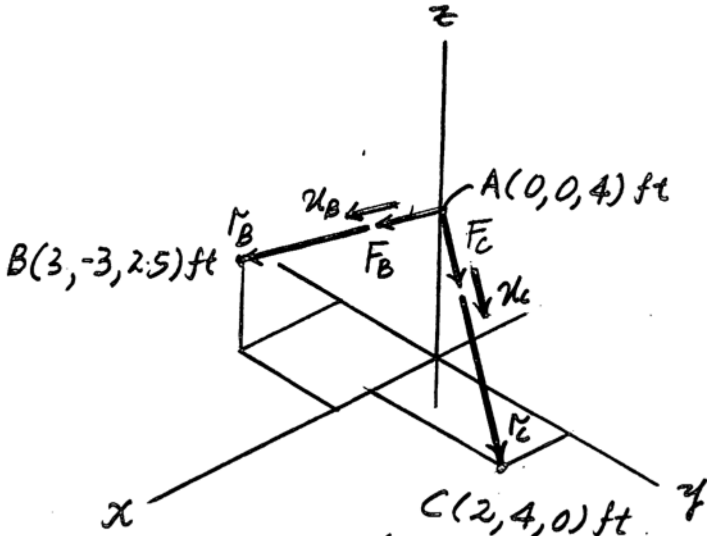
$$\mathbf{F}_R=\{650\mathbf{i}+100\mathbf{j}-700\mathbf{k}\}\text{ lb}$$

$$F_R=\sqrt{650^2+100^2+(-700)^2}=960\text{ lb} \qquad \textbf{Ans.}$$

$$\alpha=\cos^{-1}\left[\frac{(F_R)_x}{F_R}\right]=\cos^{-1}\left(\frac{650}{960}\right)=47.4^\circ \qquad \textbf{Ans.}$$

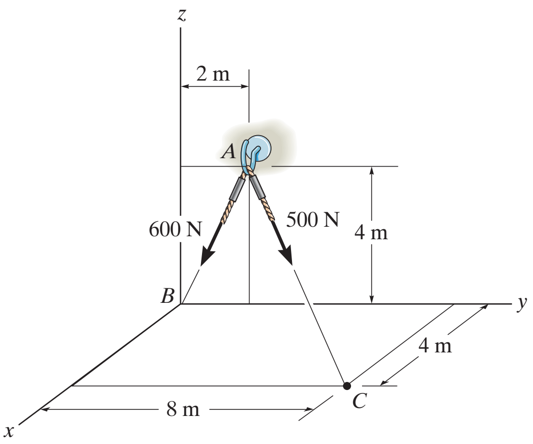
$$\beta=\cos^{-1}\left[\frac{(F_R)_y}{F_R}\right]=\cos^{-1}\left(\frac{100}{960}\right)=84.0^\circ \qquad \textbf{Ans.}$$

$$\gamma=\cos^{-1}\left[\frac{(F_R)_z}{F_R}\right]=\cos^{-1}\left(\frac{-700}{960}\right)=137^\circ \qquad \textbf{Ans.}$$



(a)

2–90. Determine the magnitude and coordinate direction angles of the resultant force.



$$\mathbf{r}_{AB} = \{-2\,\mathbf{j} - 4\,\mathbf{k}\}\text{m}; \qquad r_{AB} = 4.472\text{ m}$$

$$\mathbf{u}_{AB} = \left(\frac{\mathbf{r}_{AB}}{r_{AB}}\right) = -0.447\,\mathbf{j} - 0.894\,\mathbf{k}$$

$$\mathbf{F}_{AB} = 600\,\mathbf{u}_{AB} = \{-268.33\,\mathbf{j} - 536.66\,\mathbf{k}\}\text{N}$$

$$\mathbf{r}_{AC} = \{4\,\mathbf{i} + 6\,\mathbf{j} - 4\,\mathbf{k}\}\text{m}; \qquad r_{AC} = 8.246\text{ m}$$

$$\mathbf{u}_{AC} = \left(\frac{\mathbf{r}_{AC}}{r_{AC}}\right) = 0.485\,\mathbf{i} + 0.728\,\mathbf{j} - 0.485\,\mathbf{k}$$

$$\mathbf{F}_{AC} = 500\,\mathbf{u}_{AC} = \{242.54\,\mathbf{i} + 363.80\,\mathbf{j} - 242.54\,\mathbf{k}\}\text{N}$$

$$\mathbf{F}_R = \mathbf{F}_{AB} + \mathbf{F}_{AC}$$

$$\mathbf{F}_R = \{242.54\,\mathbf{i} + 95.47\,\mathbf{j} - 779.20\,\mathbf{k}\}$$

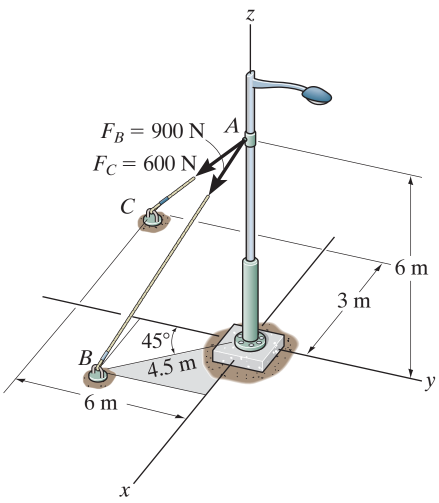
$$F_R = \sqrt{(242.54)^2 + (95.47)^2 + (-779.20)^2} = 821.64 = 822\text{ N} \qquad \textbf{Ans}$$

$$\alpha = \cos^{-1}\left(\frac{242.54}{821.64}\right) = 72.8^\circ \qquad \textbf{Ans}$$

$$\beta = \cos^{-1}\left(\frac{95.47}{821.64}\right) = 83.3^\circ \qquad \textbf{Ans}$$

$$\gamma = \cos^{-1}\left(\frac{-779.20}{821.64}\right) = 162^\circ \qquad \textbf{Ans}$$

2–91. Determine the magnitude and coordinate direction angles of the resultant force acting at *A*.



Force Vectors: The unit vectors $\mathbf{u}_B$ and $\mathbf{u}_C$ of $\mathbf{F}_B$ and $\mathbf{F}_C$ must be determined first. From Fig. *a*

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(4.5\sin 45° - 0)\mathbf{i} + (-4.5\cos 45° - 0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(4.5\sin 45° - 0)^2 + (-4.5\cos 45° - 0)^2 + (0-6)^2}}$$

$$= 0.4243\mathbf{i} - 0.4243\mathbf{j} - 0.8\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(-3-0)\mathbf{i} + (-6-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(-3-0)^2 + (-6-0)^2 + (0-6)^2}}$$

$$= -\frac{1}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} - \frac{2}{3}\mathbf{k}$$

Thus, the force vectors $\mathbf{F}_B$ and $\mathbf{F}_C$ are given by

$$\mathbf{F}_B = F_B\mathbf{u}_B = 900(0.4243\mathbf{i} - 0.4243\mathbf{j} - 0.8\mathbf{k}) = \{381.84\mathbf{i} - 381.84\mathbf{j} - 720\mathbf{k}\}\text{N}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = 600\left(-\frac{1}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} - \frac{2}{3}\mathbf{k}\right) = \{-200\mathbf{i} - 400\mathbf{j} - 400\mathbf{k}\}\text{N}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_C = (381.84\mathbf{i} - 381.84\mathbf{j} - 720\mathbf{k}) + (-200\mathbf{i} - 400\mathbf{j} - 400\mathbf{k})$$

$$= \{181.84\mathbf{i} - 781.84\mathbf{j} - 1120\mathbf{k}\}\text{ N}$$

The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2 + (F_R)_z{}^2}$$

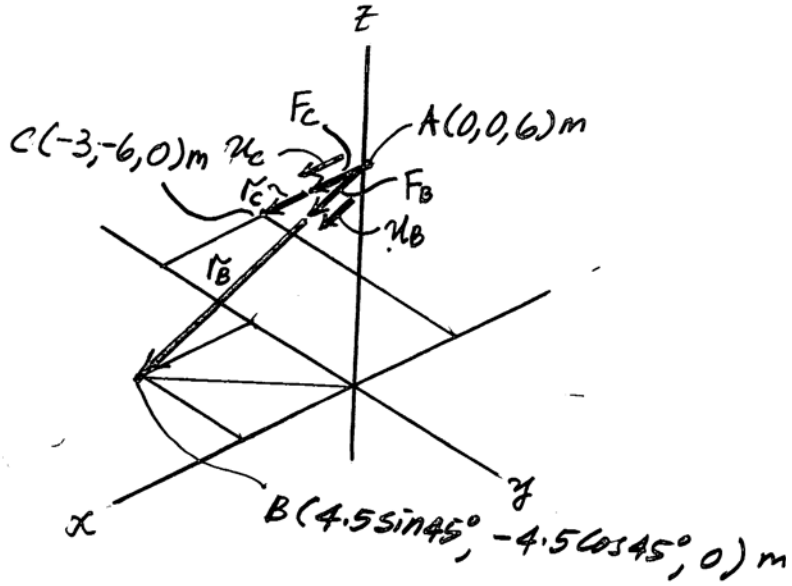
$$= \sqrt{(181.84)^2 + (-781.84)^2 + (-1120)^2} = 1377.95\text{ N} = 1.38\text{ kN} \qquad \textbf{Ans.}$$

The coordinate direction angles of $\mathbf{F}_R$ are

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{181.84}{1377.95}\right) = 82.4° \qquad \textbf{Ans.}$$

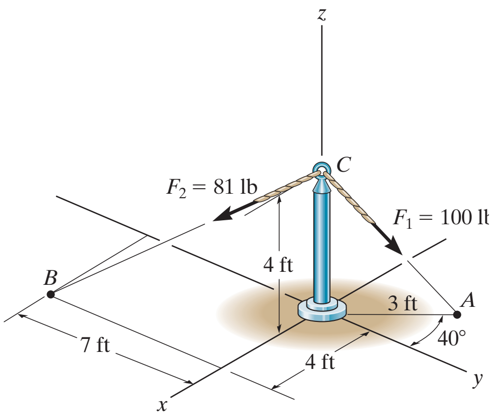
$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{-781.84}{1377.95}\right) = 125° \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{-1120}{1377.95}\right) = 144° \qquad \textbf{Ans.}$$



(a)

***2–92.** Determine the magnitude and coordinate direction angles of the resultant force.



$$\mathbf{F}_1 = -100\left(\frac{3}{5}\right)\sin 40°\,\mathbf{i} + 100\left(\frac{3}{5}\right)\cos 40°\,\mathbf{j} - 100\left(\frac{4}{5}\right)\mathbf{k}$$

$$= \{-38.567\,\mathbf{i} + 45.963\,\mathbf{j} - 80\,\mathbf{k}\}\text{ lb}$$

$$\mathbf{F}_2 = 81\text{ lb}\left(\frac{4}{9}\,\mathbf{i} - \frac{7}{9}\,\mathbf{j} - \frac{4}{9}\,\mathbf{k}\right)$$

$$= \{36\,\mathbf{i} - 63\,\mathbf{j} - 36\,\mathbf{k}\}\text{lb}$$

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 = \{-2.567\,\mathbf{i} - 17.04\,\mathbf{j} - 116.0\,\mathbf{k}\}\text{ lb}$$

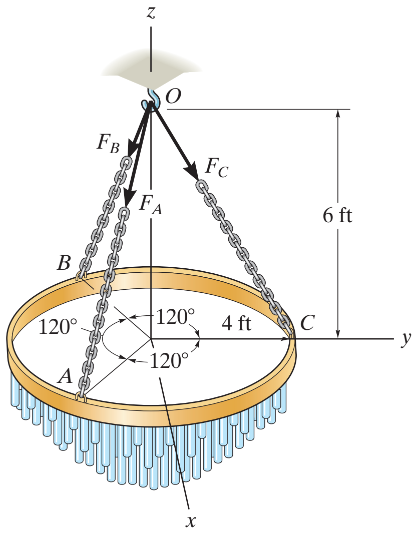
$$F_R = \sqrt{(-2.567)^2 + (-17.04)^2 + (-116.0)^2} = 117.27\text{ lb} = 117\text{ lb} \qquad \textbf{Ans}$$

$$\alpha = \cos^{-1}\left(\frac{-2.567}{117.27}\right) = 91.3° \qquad \textbf{Ans}$$

$$\beta = \cos^{-1}\left(\frac{-17.04}{117.27}\right) = 98.4° \qquad \textbf{Ans}$$

$$\gamma = \cos^{-1}\left(\frac{-116.0}{117.27}\right) = 172° \qquad \textbf{Ans}$$

•2–93. The chandelier is supported by three chains which are concurrent at point *O*. If the force in each chain has a magnitude of 60 lb, express each force as a Cartesian vector and determine the magnitude and coordinate direction angles of the resultant force.



$$\mathbf{F}_A = 60\,\frac{(4\cos 30°\,\mathbf{i} - 4\sin 30°\,\mathbf{j} - 6\,\mathbf{k})}{\sqrt{(4\cos 30°)^2 + (-4\sin 30°)^2 + (-6)^2}}$$

$$= \{28.8\,\mathbf{i} - 16.6\,\mathbf{j} - 49.9\,\mathbf{k}\}\text{ lb} \qquad \textbf{Ans}$$

$$\mathbf{F}_B = 60\,\frac{(-4\cos 30°\,\mathbf{i} - 4\sin 30°\,\mathbf{j} - 6\,\mathbf{k})}{\sqrt{(-4\cos 30°)^2 + (-4\sin 30°)^2 + (-6)^2}}$$

$$= \{-28.8\,\mathbf{i} - 16.6\,\mathbf{j} - 49.9\,\mathbf{k}\}\text{ lb} \qquad \textbf{Ans}$$

$$\mathbf{F}_C = 60\,\frac{(4\,\mathbf{j} - 6\,\mathbf{k})}{\sqrt{(4)^2 + (-6)^2}}$$

$$= \{33.3\,\mathbf{j} - 49.9\,\mathbf{k}\}\text{ lb} \qquad \textbf{Ans}$$

$$\mathbf{F}_R = \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C = \{-149.8\,\mathbf{k}\}\text{ lb}$$

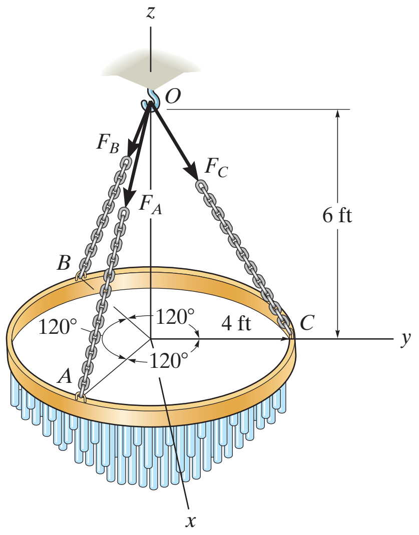
$$F_R = 150\text{ lb} \qquad \textbf{Ans}$$

$$\alpha = 90° \qquad \textbf{Ans}$$

$$\beta = 90° \qquad \textbf{Ans}$$

$$\gamma = 180° \qquad \textbf{Ans}$$

2–94. The chandelier is supported by three chains which are concurrent at point O. If the resultant force at O has a magnitude of 130 lb and is directed along the negative z axis, determine the force in each chain.



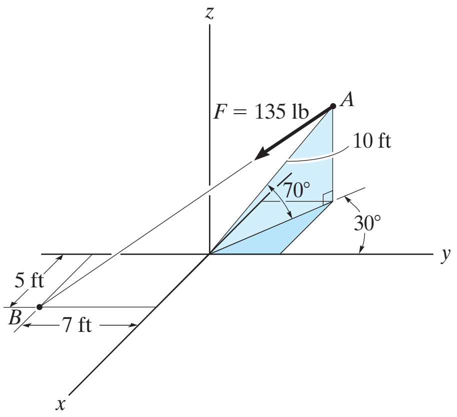
$$\mathbf{F}_C = F\frac{(4\,\mathbf{j} - 6\,\mathbf{k})}{\sqrt{4^2 + (-6)^2}} = 0.5547\,F\,\mathbf{j} - 0.8321\,F\,\mathbf{k}$$

$$\mathbf{F}_A = \mathbf{F}_B = \mathbf{F}_C$$

$F_{R_z} = \Sigma F_z$; $130 = 3(0.8321\,F)$

$F = 52.1$ lb **Ans**

2–95. Express force **F** as a Cartesian vector; then determine its coordinate direction angles.



Unit Vector : The coordinates of point A are

$$A(-10\cos 70^\circ \sin 30^\circ,\ 10\cos 70^\circ \cos 30^\circ,\ 10\sin 70^\circ)\ \text{ft}$$
$$= A(-1.710,\ 2.962,\ 9.397)\ \text{ft}$$

Then

$$\mathbf{r}_{AB} = \{[5-(-1.710)]\mathbf{i} + (-7-2.962)\mathbf{j} + (0-9.397)\mathbf{k}\}\ \text{ft}$$
$$= \{6.710\mathbf{i} - 9.962\mathbf{j} - 9.397\mathbf{k}\}\ \text{ft}$$
$$r_{AB} = \sqrt{6.710^2 + (-9.962)^2 + (-9.397)^2} = 15.250\ \text{ft}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{6.710\mathbf{i} - 9.962\mathbf{j} - 9.397\mathbf{k}}{15.250}$$
$$= 0.4400\mathbf{i} - 0.6532\mathbf{j} - 0.6162\mathbf{k}$$

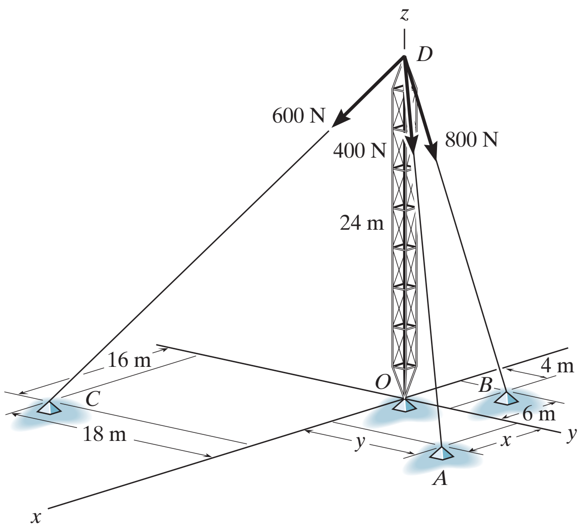
Force Vector :

$$\mathbf{F} = F\mathbf{u}_{AB} = 135\{0.4400\mathbf{i} - 0.6532\mathbf{j} - 0.6162\mathbf{k}\}\ \text{lb}$$
$$= \{59.4\mathbf{i} - 88.2\mathbf{j} - 83.2\mathbf{k}\}\ \text{lb} \qquad \textbf{Ans}$$

Coordinate Direction Angles : From the unit vector $\mathbf{u}_{AB}$ obtained above, we have

$\cos\alpha = 0.4400$ $\alpha = 63.9^\circ$ **Ans**
$\cos\beta = -0.6532$ $\beta = 131^\circ$ **Ans**
$\cos\gamma = -0.6162$ $\gamma = 128^\circ$ **Ans**

***2–96.** The tower is held in place by three cables. If the force of each cable acting on the tower is shown, determine the magnitude and coordinate direction angles α, β, γ of the resultant force. Take $x = 20$ m, $y = 15$ m.



$$\mathbf{F}_{DA} = 400\left(\frac{20}{34.66}\mathbf{i} + \frac{15}{34.66}\mathbf{j} - \frac{24}{34.66}\mathbf{k}\right) \text{ N}$$

$$\mathbf{F}_{DB} = 800\left(\frac{-6}{25.06}\mathbf{i} + \frac{4}{25.06}\mathbf{j} - \frac{24}{25.06}\mathbf{k}\right) \text{ N}$$

$$\mathbf{F}_{DC} = 600\left(\frac{16}{34}\mathbf{i} - \frac{18}{34}\mathbf{j} - \frac{24}{34}\mathbf{k}\right) \text{ N}$$

$$\mathbf{F}_R = \mathbf{F}_{DA} + \mathbf{F}_{DB} + \mathbf{F}_{DC}$$

$$= \{321.66\mathbf{i} - 16.82\mathbf{j} - 1466.71\mathbf{k}\} \text{ N}$$

$$F_R = \sqrt{(321.66)^2 + (-16.82)^2 + (-1466.71)^2}$$

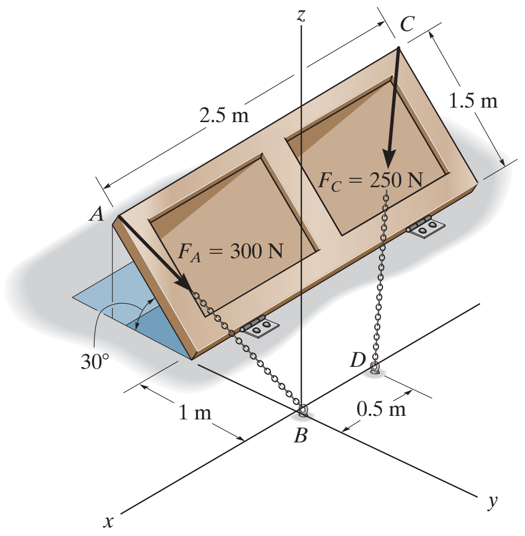
$$= 1501.66 \text{ N} = 1.50 \text{ kN} \qquad \textbf{Ans}$$

$$\alpha = \cos^{-1}\left(\frac{321.66}{1501.66}\right) = 77.6° \qquad \textbf{Ans}$$

$$\beta = \cos^{-1}\left(\frac{-16.82}{1501.66}\right) = 90.6° \qquad \textbf{Ans}$$

$$\gamma = \cos^{-1}\left(\frac{-1466.71}{1501.66}\right) = 168° \qquad \textbf{Ans}$$

•2–97. The door is held opened by means of two chains. If the tension in AB and CD is $F_A = 300$ N and $F_C = 250$ N, respectively, express each of these forces in Cartesian vector form.



Unit Vector : First determine the position vector $\mathbf{r}_{AB}$ and $\mathbf{r}_{CD}$. The coordinates of points A and C are

$$A[0,\ -(1+1.5\cos 30°),\ 1.5\sin 30°]\ \text{m} = A(0,\ -2.299,\ 0.750)\ \text{m}$$
$$C[-2.50,\ -(1+1.5\cos 30°),\ 1.5\sin 300°]\ \text{m} = C(-2.50,\ -2.299,\ 0.750)\ \text{m}$$

Then

$$\mathbf{r}_{AB} = \{(0-0)\mathbf{i}+[0-(-2.299)]\mathbf{j}+(0-0.750)\mathbf{k}\}\ \text{m}$$
$$= \{2.299\mathbf{j}-0.750\mathbf{k}\}\ \text{m}$$
$$r_{AB} = \sqrt{2.299^2+(-0.750)^2} = 2.418\ \text{m}$$
$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{2.299\mathbf{j}-0.750\mathbf{k}}{2.418} = 0.9507\mathbf{j}-0.3101\mathbf{k}$$

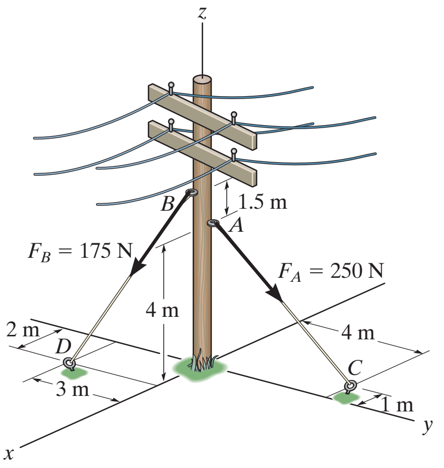
$$\mathbf{r}_{CD} = \{[-0.5-(-2.5)]\mathbf{i}+[0-(-2.299)]\mathbf{j}+(0-0.750)\mathbf{k}\}\ \text{m}$$
$$= \{2.00\mathbf{i}+2.299\mathbf{j}-0.750\mathbf{k}\}\ \text{m}$$
$$r_{CD} = \sqrt{2.00^2+2.299^2+(-0.750)^2} = 3.138\ \text{m}$$
$$\mathbf{u}_{CD} = \frac{\mathbf{r}_{CD}}{r_{CD}} = \frac{2.00\mathbf{i}+2.299\mathbf{j}-0.750\mathbf{k}}{3.138} = 0.6373\mathbf{i}+0.7326\mathbf{j}-0.2390\mathbf{k}$$

Force Vector :

$$\mathbf{F}_A = F_A\mathbf{u}_{AB} = 300\{0.9507\mathbf{j}-0.3101\mathbf{k}\}\ \text{N}$$
$$= \{285.21\mathbf{j}-93.04\mathbf{k}\}\ \text{N}$$
$$= \{285\mathbf{j}-93.0\mathbf{k}\}\ \text{N} \qquad \textbf{Ans}$$

$$\mathbf{F}_C = F_C\mathbf{u}_{CD} = 250\{0.6373\mathbf{i}+0.7326\mathbf{j}-0.2390\mathbf{k}\}\ \text{N}$$
$$= \{159.33\mathbf{i}+183.15\mathbf{j}-59.75\mathbf{k}\}\ \text{N}$$
$$= \{159\mathbf{i}+183\mathbf{j}-59.7\mathbf{k}\}\ \text{N} \qquad \textbf{Ans}$$

2–98. The guy wires are used to support the telephone pole. Represent the force in each wire in Cartesian vector form. Neglect the diameter of the pole.



Unit Vector :

$$\mathbf{r}_{AC} = \{(-1-0)\mathbf{i}+(4-0)\mathbf{j}+(0-4)\mathbf{k}\}\ \text{m} = \{-1\mathbf{i}+4\mathbf{j}-4\mathbf{k}\}\ \text{m}$$
$$r_{AC} = \sqrt{(-1)^2+4^2+(-4)^2} = 5.745\ \text{m}$$
$$\mathbf{u}_{AC} = \frac{\mathbf{r}_{AC}}{r_{AC}} = \frac{-1\mathbf{i}+4\mathbf{j}-4\mathbf{k}}{5.745} = -0.1741\mathbf{i}+0.6963\mathbf{j}-0.6963\mathbf{k}$$

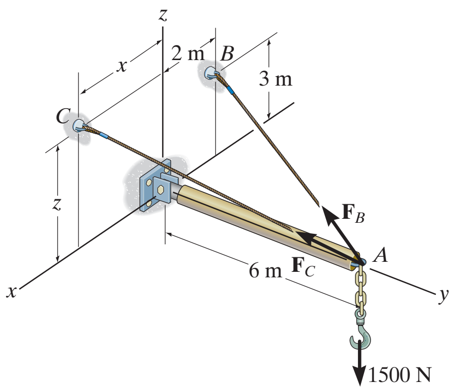
$$\mathbf{r}_{BD} = \{(2-0)\mathbf{i}+(-3-0)\mathbf{j}+(0-5.5)\mathbf{k}\}\ \text{m} = \{2\mathbf{i}-3\mathbf{j}-5.5\mathbf{k}\}\ \text{m}$$
$$r_{BD} = \sqrt{2^2+(-3)^2+(-5.5)^2} = 6.576\ \text{m}$$
$$\mathbf{u}_{BD} = \frac{\mathbf{r}_{BD}}{r_{BD}} = \frac{2\mathbf{i}-3\mathbf{j}-5.5\mathbf{k}}{6.576} = 0.3041\mathbf{i}-0.4562\mathbf{j}-0.8363\mathbf{k}$$

Force Vector :

$$\mathbf{F}_A = F_A\mathbf{u}_{AC} = 250\{-0.1741\mathbf{i}+0.6963\mathbf{j}-0.6963\mathbf{k}\}\ \text{N}$$
$$= \{-43.52\mathbf{i}+174.08\mathbf{j}-174.08\mathbf{k}\}\ \text{N}$$
$$= \{-43.5\mathbf{i}+174\mathbf{j}-174\mathbf{k}\}\ \text{N} \qquad \textbf{Ans}$$

$$\mathbf{F}_B = F_B\mathbf{u}_{BD} = 175\{0.3041\mathbf{i}-0.4562\mathbf{j}-0.8363\mathbf{k}\}\ \text{N}$$
$$= \{53.22\mathbf{i}-79.83\mathbf{j}-146.36\mathbf{k}\}\ \text{N}$$
$$= \{53.2\mathbf{i}-79.8\mathbf{j}-146\mathbf{k}\}\ \text{N} \qquad \textbf{Ans}$$

2–99. Two cables are used to secure the overhang boom in position and support the 1500-N load. If the resultant force is directed along the boom from point A towards O, determine the magnitudes of the resultant force and forces $\mathbf{F}_B$ and $\mathbf{F}_C$. Set $x = 3$ m and $z = 2$ m.



Force Vectors: The unit vectors $\mathbf{u}_B$ and $\mathbf{u}_C$ must be determined first. From Fig. *a*,

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(-2-0)\mathbf{i}+(0-6)\mathbf{j}+(3-0)\mathbf{k}}{\sqrt{(-2-0)^2+(0-6)^2+(3-0)^2}} = -\frac{2}{7}\mathbf{i}-\frac{6}{7}\mathbf{j}+\frac{3}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(3-0)\mathbf{i}+(0-6)\mathbf{j}+(2-0)\mathbf{k}}{\sqrt{(3-0)^2+(0-6)^2+(2-0)^2}} = \frac{3}{7}\mathbf{i}-\frac{6}{7}\mathbf{j}+\frac{2}{7}\mathbf{k}$$

Thus, the force vectors $\mathbf{F}_B$ and $\mathbf{F}_C$ are given by

$$\mathbf{F}_B = F_B\mathbf{u}_B = -\frac{2}{7}F_B\mathbf{i}-\frac{6}{7}F_B\mathbf{j}+\frac{3}{7}F_B\mathbf{k}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = \frac{3}{7}F_C\mathbf{i}-\frac{6}{7}F_C\mathbf{j}+\frac{2}{7}F_C\mathbf{k}$$

Since the resultant force $\mathbf{F}_R$ is directed along the negative y axis, and the load $\mathbf{W}$ is directed along the z axis, these two forces can be written as

$$\mathbf{F}_R = -F_R\mathbf{j} \qquad \text{and} \qquad \mathbf{W} = [-1500\mathbf{k}]\ \text{N}$$

Resultant Force: The vector addition of $\mathbf{F}_B$, $\mathbf{F}_C$, and $\mathbf{W}$ is equal to $\mathbf{F}_R$. Thus,

$$\mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_C + \mathbf{W}$$

$$-F_R\mathbf{j} = \left(-\frac{2}{7}F_B\mathbf{i}-\frac{6}{7}F_B\mathbf{j}+\frac{3}{7}F_B\mathbf{k}\right)+\left(\frac{3}{7}F_C\mathbf{i}-\frac{6}{7}F_C\mathbf{j}+\frac{2}{7}F_C\mathbf{k}\right)+(-1500\mathbf{k})$$

$$-F_R\mathbf{j} = \left(-\frac{2}{7}F_B+\frac{3}{7}F_C\right)\mathbf{i}+\left(-\frac{6}{7}F_B-\frac{6}{7}F_C\right)\mathbf{j}+\left(\frac{3}{7}F_B+\frac{2}{7}F_C-1500\right)\mathbf{k}$$

Equating the **i**, **j**, and **k** components,

$$0 = -\frac{2}{7}F_B+\frac{3}{7}F_C \qquad (1)$$

$$-F_R = -\frac{6}{7}F_B-\frac{6}{7}F_C \qquad (2)$$

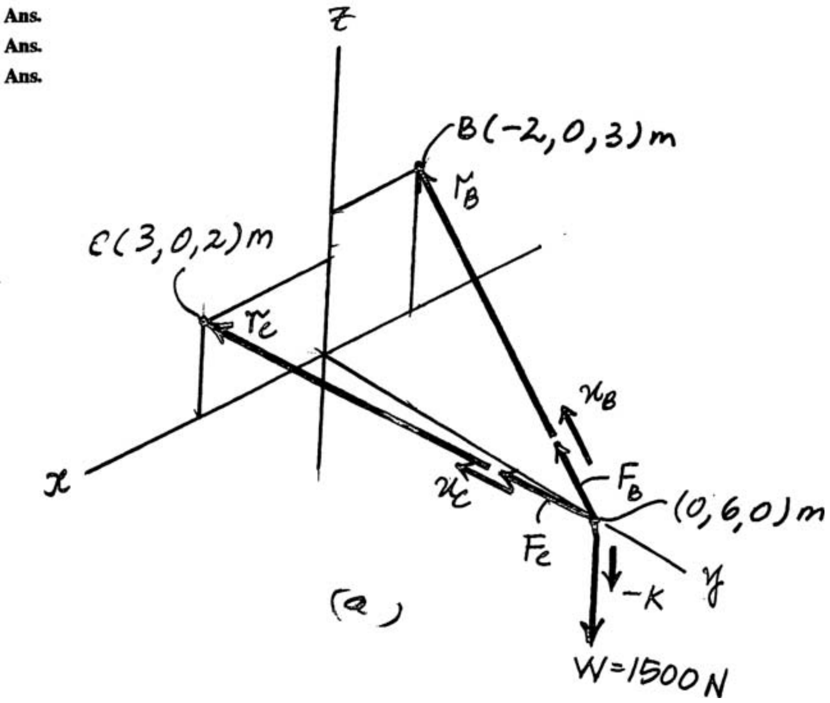
$$0 = \frac{3}{7}F_B+\frac{2}{7}F_C-1500 \qquad (3)$$

Solving Eqs. (1), (2), and (3) yields

$F_C = 1615.38\ \text{N} = 1.62\ \text{kN}$ **Ans.**

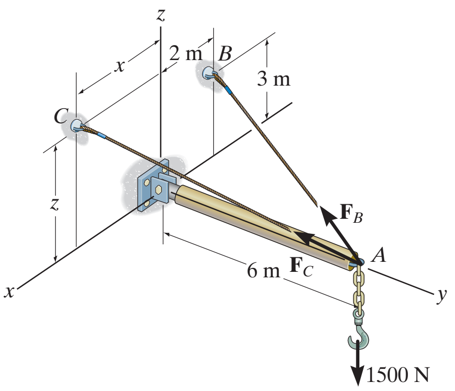
$F_B = 2423.08\ \text{N} = 2.42\ \text{kN}$ **Ans.**

$F_R = 3461.53\ \text{N} = 3.46\ \text{kN}$ **Ans.**



(a)

***2–100.** Two cables are used to secure the overhang boom in position and support the 1500-N load. If the resultant force is directed along the boom from point A towards O, determine the values of x and z for the coordinates of point C and the magnitude of the resultant force. Set $F_B = 1610$ N and $F_C = 2400$ N.



Force Vectors: From Fig. *a*,

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(-2-0)\mathbf{i}+(0-6)\mathbf{j}+(3-0)\mathbf{k}}{\sqrt{(-2-0)^2+(0-6)^2+(3-0)^2}} = -\frac{2}{7}\mathbf{i}-\frac{6}{7}\mathbf{j}+\frac{3}{7}\mathbf{k}$$

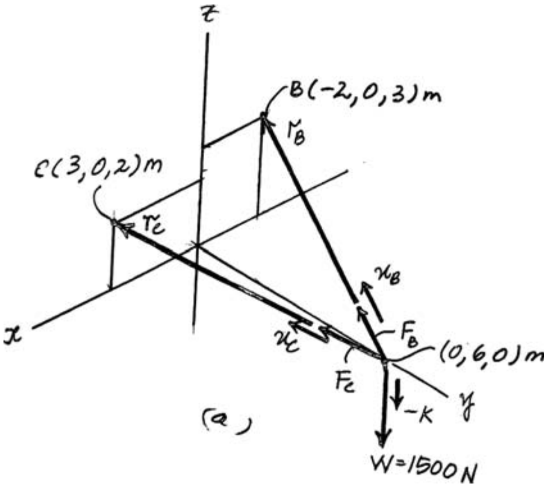
$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(x-0)\mathbf{i}+(0-6)\mathbf{j}+(z-0)\mathbf{k}}{\sqrt{(x-0)^2+(0-6)^2+(z-0)^2}} = \frac{x}{\sqrt{x^2+z^2+36}}\mathbf{i}-\frac{6}{\sqrt{x^2+z^2+36}}\mathbf{j}+\frac{z}{\sqrt{x^2+z^2+36}}\mathbf{k}$$

Thus,

$$\mathbf{F}_B = F_B\mathbf{u}_B = 1610\left(-\frac{2}{7}\mathbf{i}-\frac{6}{7}\mathbf{j}+\frac{3}{7}\mathbf{k}\right) = [-460\mathbf{i}-1380\mathbf{j}+690\mathbf{k}]\text{N}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = 2400\left(\frac{x}{\sqrt{x^2+z^2+36}}\mathbf{i}-\frac{6}{\sqrt{x^2+z^2+36}}\mathbf{j}+\frac{z}{\sqrt{x^2+z^2+36}}\mathbf{k}\right)$$

$$= \frac{2400x}{\sqrt{x^2+z^2+36}}\mathbf{i}-\frac{14\,400}{\sqrt{x^2+z^2+36}}\mathbf{j}+\frac{2400z}{\sqrt{x^2+z^2+36}}$$



Since the resultant force $\mathbf{F}_R$ is directed along the negative y axis, and the load is directed along the z axis, these two forces can be written as

$$\mathbf{F}_R = -F_R\mathbf{j} \qquad \text{and} \qquad \mathbf{W} = [-1500\mathbf{k}]\text{ N}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_C + \mathbf{W}$$

$$-F_R\mathbf{j} = (-460\mathbf{i}-1380\mathbf{j}+690\mathbf{k}) + \left(\frac{2400x}{\sqrt{x^2+z^2+36}}\mathbf{i}-\frac{14\,400}{\sqrt{x^2+z^2+36}}\mathbf{j}+\frac{2400z}{\sqrt{x^2+z^2+36}}\mathbf{k}\right)+(-1500\mathbf{k})$$

$$-F_R\mathbf{j} = \left(\frac{2400x}{\sqrt{x^2+z^2+36}}-460\right)\mathbf{i}-\left(\frac{14\,400}{\sqrt{x^2+z^2+36}}+1380\right)\mathbf{j}+\left(690+\frac{2400z}{\sqrt{x^2+z^2+36}}-1500\right)\mathbf{k}$$

Equating the **i**, **j**, and **k** components,

$$0 = \frac{2400x}{\sqrt{x^2+z^2+36}}-460 \qquad \frac{2400x}{\sqrt{x^2+z^2+36}} = 460 \qquad (1)$$

$$-F_R = -\left(\frac{14400}{\sqrt{x^2+z^2+36}}+1380\right) \qquad F_R = \frac{14400}{\sqrt{x^2+z^2+36}}+1380 \qquad (2)$$

$$0 = 690+\frac{2400z}{\sqrt{x^2+z^2+36}}-1500 \qquad \frac{2400z}{\sqrt{x^2+z^2+36}} = 810 \qquad (3)$$

Dividing Eq. (1) by Eq. (3), yields

$$x = 0.5679z \qquad (4)$$

Substituting Eq. (4) into Eq. (1), and solving

$z = 2.197\text{ m} = 2.20\text{ m}$ **Ans.**

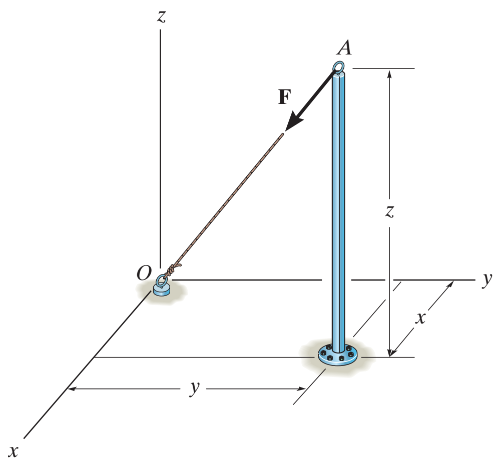
Substituting $z = 2.197$ m into Eq. (4), yields

$x = 1.248\text{ m} = 1.25\text{ m}$ **Ans.**

Substituting $x = 1.248$ m and $z = 2.197$ m into Eq. (2), yields

$F_R = 3591.85\text{ N} = 3.59\text{ kN}$ **Ans.**

•2–101. The cable *AO* exerts a force on the top of the pole of $\mathbf{F} = \{-120\mathbf{i} - 90\mathbf{j} - 80\mathbf{k}\}$ lb. If the cable has a length of 34 ft, determine the height z of the pole and the location (x, y) of its base.



$$F = \sqrt{(-120)^2 + (-90)^2 + (-80)^2} = 170 \text{ lb}$$

$$\mathbf{u} = \frac{\mathbf{F}}{F} = -\frac{120}{170}\mathbf{i} - \frac{90}{170}\mathbf{j} - \frac{80}{170}\mathbf{k}$$

$$\mathbf{r} = 34\,\mathbf{u} = \{-24\,\mathbf{i} - 18\,\mathbf{j} - 16\,\mathbf{k}\}\text{ ft}$$

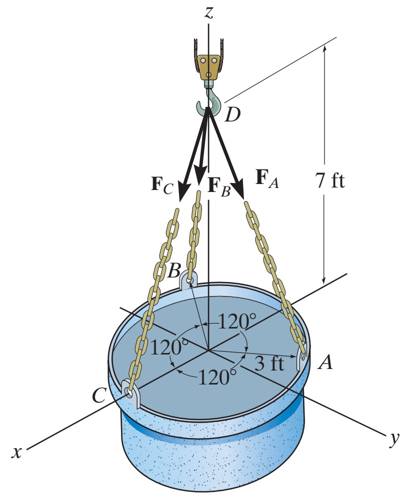
Thus,

$x = 24$ ft **Ans**

$y = 18$ ft **Ans**

$z = 16$ ft **Ans**

2–102. If the force in each chain has a magnitude of 450 lb, determine the magnitude and coordinate direction angles of the resultant force.



Force Vectors: The unit vectors $\mathbf{u}_A$, $\mathbf{u}_B$, and $\mathbf{u}_C$ must be determined first. From Fig. *a*,

$$\mathbf{u}_A = \frac{(-3\sin 30° - 0)\mathbf{i} + (3\cos 30° - 0)\mathbf{j} + (0-7)\mathbf{k}}{\sqrt{(-3\sin 30° - 0)^2 + (3\cos 30° - 0)^2 + (0-7)^2}} = -0.1970\mathbf{i} + 0.3411\mathbf{j} - 0.9191\mathbf{k}$$

$$\mathbf{u}_B = \frac{(-3\sin 30° - 0)\mathbf{i} + (-3\cos 30° - 0)\mathbf{j} + (0-7)\mathbf{k}}{\sqrt{(-3\sin 30° - 0)^2 + (-3\cos 30° - 0)^2 + (0-7)^2}} = -0.1970\mathbf{i} - 0.3411\mathbf{j} - 0.9191\mathbf{k}$$

$$\mathbf{u}_C = \frac{(3-0)\mathbf{i} + (0-0)\mathbf{j} + (0-7)\mathbf{k}}{\sqrt{(3-0)^2 + (0-0)^2 + (0-7)^2}} = 0.3939\mathbf{i} - 0.9191\mathbf{k}$$

Thus, the force vectors $\mathbf{F}_A$, $\mathbf{F}_B$, and $\mathbf{F}_C$ are given by

$$\mathbf{F}_A = F_A\mathbf{u}_A = 450(-0.1970\mathbf{i} + 0.3411\mathbf{j} - 0.9191\mathbf{k}) = \{-88.63\mathbf{i} + 153.51\mathbf{j} - 413.62\mathbf{k}\}\text{ lb}$$

$$\mathbf{F}_B = F_B\mathbf{u}_B = 450(-0.1970\mathbf{i} - 0.3411\mathbf{j} - 0.9191\mathbf{k}) = \{-88.63\mathbf{i} - 153.51\mathbf{j} - 413.62\mathbf{k}\}\text{ lb}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = 450(0.3939\mathbf{i} - 0.9191\mathbf{k}) = \{177.26\mathbf{i} - 413.62\mathbf{k}\}\text{ lb}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C = (-88.63\mathbf{i} + 153.51\mathbf{j} - 413.62\mathbf{k}) + (-88.63\mathbf{i} - 153.51\mathbf{j} - 413.62\mathbf{k}) + (177.26\mathbf{i} - 413.62\mathbf{k})$$
$$= \{-1240.85\mathbf{k}\}\text{ lb}$$

The magnitude of $\mathbf{F}_R$ is

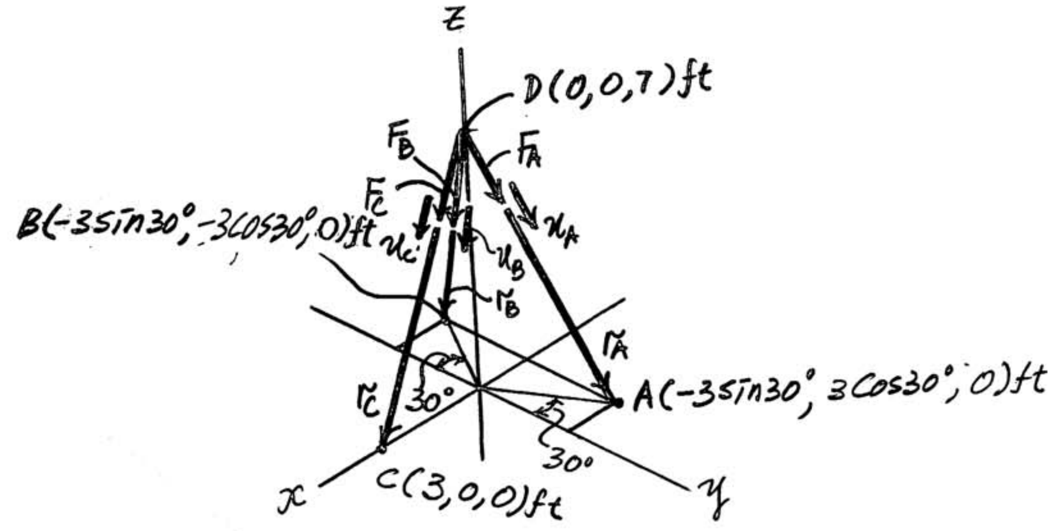
$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2 + (F_R)_z{}^2}$$
$$= \sqrt{0^2 + 0^2 + (-1240.85)^2} = 1240.85\text{ lb} = 1.24\text{ kip} \qquad \textbf{Ans.}$$

The coordinate direction angles of $\mathbf{F}_R$ are

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{0}{1240.85}\right) = 90° \qquad \textbf{Ans.}$$

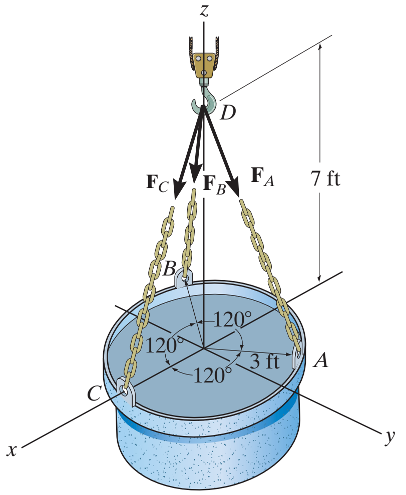
$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{0}{1240.85}\right) = 90° \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{-1240.85}{1240.85}\right) = 180° \qquad \textbf{Ans.}$$



(a)

2–103. If the resultant of the three forces is $\mathbf{F}_R = \{-900\mathbf{k}\}$ lb, determine the magnitude of the force in each chain.



Force Vectors: The unit vectors $\mathbf{u}_A$, $\mathbf{u}_B$, and $\mathbf{u}_C$ must be determined first. From Fig. a,

$$\mathbf{u}_A = \frac{(-3\sin 30° - 0)\mathbf{i} + (3\cos 30° - 0)\mathbf{j} + (0-7)\mathbf{k}}{\sqrt{(-3\sin 30° - 0)^2 + (3\cos 30° - 0)^2 + (0-7)^2}} = -0.1970\mathbf{i} + 0.3411\mathbf{j} - 0.9191\mathbf{k}$$

$$\mathbf{u}_B = \frac{(-3\sin 30° - 0)\mathbf{i} + (-3\cos 30° - 0)\mathbf{j} + (0-7)\mathbf{k}}{\sqrt{(-3\sin 30° - 0)^2 + (-3\cos 30° - 0)^2 + (0-7)^2}} = -0.1970\mathbf{i} - 0.3411\mathbf{j} - 0.9191\mathbf{k}$$

$$\mathbf{u}_C = \frac{(3-0)\mathbf{i} + (0-0)\mathbf{j} + (0-7)\mathbf{k}}{\sqrt{(3-0)^2 + (0-0)^2 + (0-7)^2}} = 0.3939\mathbf{i} - 0.9191\mathbf{k}$$

Thus, the force vectors $\mathbf{F}_A$, $\mathbf{F}_B$, and $\mathbf{F}_C$ are given by

$$\mathbf{F}_A = F_A\mathbf{u}_A = -0.1970F_A\mathbf{i} + 0.3411F_A\mathbf{j} - 0.9191F_A\mathbf{k}$$
$$\mathbf{F}_B = F_B\mathbf{u}_B = -0.1970F_B\mathbf{i} - 0.3411F_B\mathbf{j} - 0.9191F_B\mathbf{k}$$
$$\mathbf{F}_C = F_C\mathbf{u}_C = 0.3939F_C\mathbf{i} - 0.9191F_C\mathbf{k}$$

Resultant Force: The vector addition of $\mathbf{F}_A$, $\mathbf{F}_B$, and $\mathbf{F}_C$ is equal to $\mathbf{F}_R$. Thus,

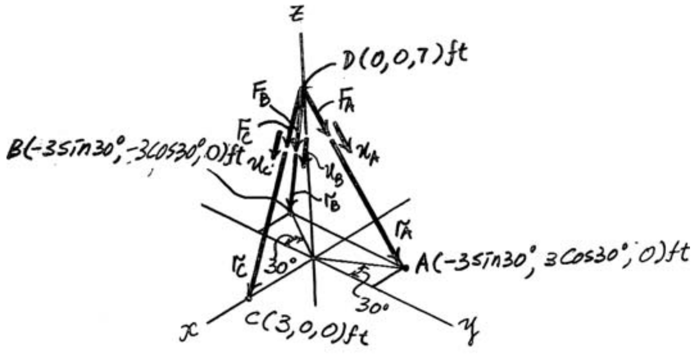
$$\mathbf{F}_R = \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C$$
$$-900\mathbf{k} = (-0.1970F_A\mathbf{i} + 0.3411F_A\mathbf{j} - 0.9191F_A\mathbf{k}) + (-0.1970F_B\mathbf{i} - 0.3411F_B\mathbf{j} - 0.9191F_B\mathbf{k}) + (0.3939F_C\mathbf{i} - 0.9191F_C\mathbf{k})$$
$$-900\mathbf{k} = (-0.1970F_A - 0.1970F_B + 0.3939F_C)\mathbf{i} + (0.3411F_A - 0.3411F_B)\mathbf{j} + (-0.9191F_A - 0.9191F_B - 0.9191F_C)\mathbf{k}$$

Equating the **i**, **j**, and **k** components,

$$0 = -0.1970F_A - 0.1970F_B + 0.3939F_C \qquad (1)$$
$$0 = 0.3411F_A - 0.3411F_B \qquad (2)$$
$$-900\mathbf{k} = -0.9191F_A - 0.9191F_B - 0.9191F_C \qquad (3)$$

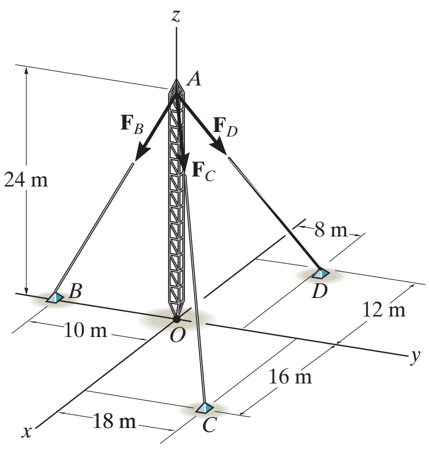
Solving Eqs. (1), (2), and (3), yields

$F_A = F_B = F_C = 326$ lb **Ans.**



(a)

***2–104.** The antenna tower is supported by three cables. If the forces of these cables acting on the antenna are $F_B = 520$ N, $F_C = 680$ N, and $F_D = 560$ N, determine the magnitude and coordinate direction angles of the resultant force acting at A.



$$\mathbf{F}_B = 520\left(\frac{\mathbf{r}_{AB}}{r_{AB}}\right) = 520\left(-\frac{10}{26}\mathbf{j} - \frac{24}{26}\mathbf{k}\right) = -200\,\mathbf{j} - 480\,\mathbf{k}$$

$$\mathbf{F}_C = 680\left(\frac{\mathbf{r}_{AC}}{r_{AC}}\right) = 680\left(\frac{16}{34}\mathbf{i} + \frac{18}{34}\mathbf{j} - \frac{24}{34}\mathbf{k}\right) = 320\,\mathbf{i} + 360\,\mathbf{j} - 480\,\mathbf{k}$$

$$\mathbf{F}_D = 560\left(\frac{\mathbf{r}_{AD}}{r_{AD}}\right) = 560\left(-\frac{12}{28}\mathbf{i} + \frac{8}{28}\mathbf{j} - \frac{24}{28}\mathbf{k}\right) = -240\,\mathbf{i} + 160\,\mathbf{j} - 480\,\mathbf{k}$$

$$\mathbf{F}_R = \Sigma\mathbf{F} = \{80\,\mathbf{i} + 320\,\mathbf{j} - 1440\,\mathbf{k}\}\text{ N}$$

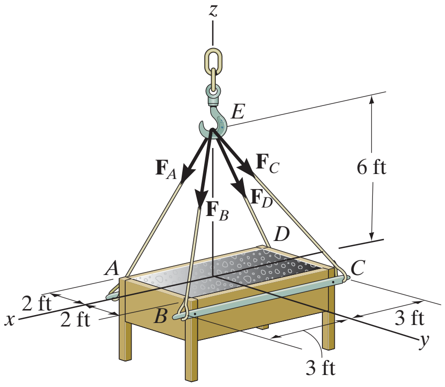
$$\mathbf{F}_R = \sqrt{(80)^2 + (320)^2 + (-1440)^2} = 1477.3 = 1.48\text{ kN} \qquad \textbf{Ans}$$

$$\alpha = \cos^{-1}\left(\frac{80}{1477.3}\right) = 86.9^\circ \qquad \textbf{Ans}$$

$$\beta = \cos^{-1}\left(\frac{320}{1477.3}\right) = 77.5^\circ \qquad \textbf{Ans}$$

$$\gamma = \cos^{-1}\left(\frac{-1440}{1477.3}\right) = 167^\circ \qquad \textbf{Ans}$$

•2–105. If the force in each cable tied to the bin is 70 lb, determine the magnitude and coordinate direction angles of the resultant force.



Force Vectors: The unit vectors $\mathbf{u}_A$, $\mathbf{u}_B$, $\mathbf{u}_C$ and $\mathbf{u}_D$ of $\mathbf{F}_A$, $\mathbf{F}_B$, $\mathbf{F}_C$ and $\mathbf{F}_D$ must be determined first. From Fig. *a*,

$$\mathbf{u}_A=\frac{\mathbf{r}_A}{r_A}=\frac{(3-0)\mathbf{i}+(-2-0)\mathbf{j}+(0-6)\mathbf{k}}{\sqrt{(3-0)^2+(-2-0)^2+(0-6)^2}}=\frac{3}{7}\mathbf{i}-\frac{2}{7}\mathbf{j}-\frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_B=\frac{\mathbf{r}_B}{r_B}=\frac{(3-0)\mathbf{i}+(2-0)\mathbf{j}+(0-6)\mathbf{k}}{\sqrt{(3-0)^2+(2-0)^2+(0-6)^2}}=\frac{3}{7}\mathbf{i}+\frac{2}{7}\mathbf{j}-\frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_C=\frac{\mathbf{r}_C}{r_C}=\frac{(-3-0)\mathbf{i}+(2-0)\mathbf{j}+(0-6)\mathbf{k}}{\sqrt{(-3-0)^2+(2-0)^2+(0-6)^2}}=-\frac{3}{7}\mathbf{i}+\frac{2}{7}\mathbf{j}-\frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_D=\frac{\mathbf{r}_D}{r_D}=\frac{(-3-0)\mathbf{i}+(-2-0)\mathbf{j}+(0-6)\mathbf{k}}{\sqrt{(-3-0)^2+(-2-0)^2+(0-6)^2}}=-\frac{3}{7}\mathbf{i}-\frac{2}{7}\mathbf{j}-\frac{6}{7}\mathbf{k}$$

Thus, the force vectors $\mathbf{F}_A$, $\mathbf{F}_B$, $\mathbf{F}_C$ and $\mathbf{F}_D$ are given by

$$\mathbf{F}_A=F_A\mathbf{u}_A=70\left(\frac{3}{7}\mathbf{i}-\frac{2}{7}\mathbf{j}-\frac{6}{7}\mathbf{k}\right)=[30\mathbf{i}-20\mathbf{j}-60\mathbf{k}]\text{ lb}$$

$$\mathbf{F}_B=F_B\mathbf{u}_B=70\left(\frac{3}{7}\mathbf{i}+\frac{2}{7}\mathbf{j}-\frac{6}{7}\mathbf{k}\right)=[30\mathbf{i}+20\mathbf{j}-60\mathbf{k}]\text{ lb}$$

$$\mathbf{F}_C=F_C\mathbf{u}_C=70\left(-\frac{3}{7}\mathbf{i}+\frac{2}{7}\mathbf{j}-\frac{6}{7}\mathbf{k}\right)=[-30\mathbf{i}+20\mathbf{j}-60\mathbf{k}]\text{ lb}$$

$$\mathbf{F}_D=F_D\mathbf{u}_D=70\left(-\frac{3}{7}\mathbf{i}-\frac{2}{7}\mathbf{j}-\frac{6}{7}\mathbf{k}\right)=[-30\mathbf{i}-20\mathbf{j}-60\mathbf{k}]\text{ lb}$$

Resultant Force:

$$\mathbf{F}_R=\mathbf{F}_A+\mathbf{F}_B+\mathbf{F}_C+\mathbf{F}_D=(30\mathbf{i}-20\mathbf{j}-60\mathbf{k})+(30\mathbf{i}+20\mathbf{j}-60\mathbf{k})+(-30\mathbf{i}+20\mathbf{j}-60\mathbf{k})+(-30\mathbf{i}-20\mathbf{j}-60\mathbf{k})$$
$$=\{-240\mathbf{k}\}\text{ N}$$

The magnitude of $\mathbf{F}_R$ is

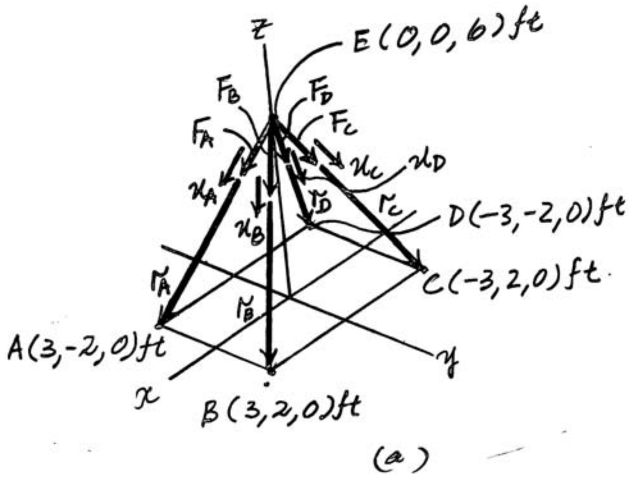
$$F_R=\sqrt{(F_R)_x{}^2+(F_R)_y{}^2+(F_R)_z{}^2}$$
$$=\sqrt{0+0+(-240)^2}=240\text{ lb} \qquad \textbf{Ans.}$$

The coordinate direction angles of $\mathbf{F}_R$ are

$$\alpha=\cos^{-1}\left[\frac{(F_R)_x}{F_R}\right]=\cos^{-1}\left(\frac{0}{240}\right)=90° \qquad \textbf{Ans.}$$

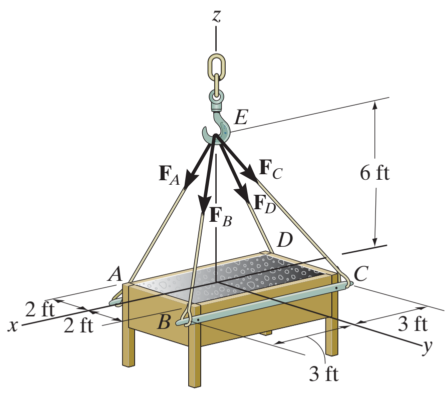
$$\beta=\cos^{-1}\left[\frac{(F_R)_y}{F_R}\right]=\cos^{-1}\left(\frac{0}{240}\right)=90° \qquad \textbf{Ans.}$$

$$\gamma=\cos^{-1}\left[\frac{(F_R)_z}{F_R}\right]=\cos^{-1}\left(\frac{-240}{240}\right)=180° \qquad \textbf{Ans.}$$



(a)

2–106. If the resultant of the four forces is $\mathbf{F}_R = \{-360\mathbf{k}\}$ lb, determine the tension developed in each cable. Due to symmetry, the tension in the four cables is the same.



Force Vectors: The unit vectors $\mathbf{u}_A$, $\mathbf{u}_B$, $\mathbf{u}_C$ and $\mathbf{u}_D$ of $\mathbf{F}_A$, $\mathbf{F}_B$, $\mathbf{F}_C$ and $\mathbf{F}_D$ must be determined first. From Fig. *a*,

$$\mathbf{u}_A = \frac{\mathbf{r}_A}{r_A} = \frac{(3-0)\mathbf{i}+(-2-0)\mathbf{j}+(0-6)\mathbf{k}}{\sqrt{(3-0)^2+(-2-0)^2+(0-6)^2}} = \frac{3}{7}\mathbf{i}-\frac{2}{7}\mathbf{j}-\frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(3-0)\mathbf{i}+(2-0)\mathbf{j}+(0-6)\mathbf{k}}{\sqrt{(3-0)^2+(2-0)^2+(0-6)^2}} = \frac{3}{7}\mathbf{i}+\frac{2}{7}\mathbf{j}-\frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(-3-0)\mathbf{i}+(2-0)\mathbf{j}+(0-6)\mathbf{k}}{\sqrt{(-3-0)^2+(2-0)^2+(0-6)^2}} = -\frac{3}{7}\mathbf{i}+\frac{2}{7}\mathbf{j}-\frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_D = \frac{\mathbf{r}_D}{r_D} = \frac{(-3-0)\mathbf{i}+(-2-0)\mathbf{j}+(0-6)\mathbf{k}}{\sqrt{(-3-0)^2+(-2-0)^2+(0-6)^2}} = -\frac{3}{7}\mathbf{i}-\frac{2}{7}\mathbf{j}-\frac{6}{7}\mathbf{k}$$

Since the magnitudes of $\mathbf{F}_A$, $\mathbf{F}_B$, $\mathbf{F}_C$ and $\mathbf{F}_D$ are the same and denoted as $\mathbf{F}$, they can be written as

$$\mathbf{F}_A = F_A\mathbf{u}_A = F\left(\frac{3}{7}\mathbf{i}-\frac{2}{7}\mathbf{j}-\frac{6}{7}\mathbf{k}\right)$$

$$\mathbf{F}_B = F_B\mathbf{u}_B = F\left(\frac{3}{7}\mathbf{i}+\frac{2}{7}\mathbf{j}-\frac{6}{7}\mathbf{k}\right)$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = F\left(-\frac{3}{7}\mathbf{i}+\frac{2}{7}\mathbf{j}-\frac{6}{7}\mathbf{k}\right)$$

$$\mathbf{F}_D = F_D\mathbf{u}_D = F\left(-\frac{3}{7}\mathbf{i}-\frac{2}{7}\mathbf{j}-\frac{6}{7}\mathbf{k}\right)$$

Resultant Force: The vector addition of $\mathbf{F}_A$, $\mathbf{F}_B$, $\mathbf{F}_C$ and $\mathbf{F}_D$ is equal to $\mathbf{F}_R$. Thus,

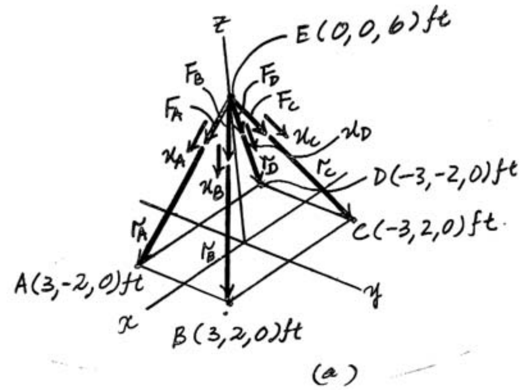
$$\mathbf{F}_R = \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D$$

$$\{-360\mathbf{k}\} = \left[F\left(\frac{3}{7}\mathbf{i}-\frac{2}{7}\mathbf{j}-\frac{6}{7}\mathbf{k}\right)\right]+\left[F\left(\frac{3}{7}\mathbf{i}+\frac{2}{7}\mathbf{j}-\frac{6}{7}\mathbf{k}\right)\right]+\left[F\left(-\frac{3}{7}\mathbf{i}+\frac{2}{7}\mathbf{j}-\frac{6}{7}\mathbf{k}\right)\right]+\left[F\left(-\frac{3}{7}\mathbf{i}-\frac{2}{7}\mathbf{j}-\frac{6}{7}\mathbf{k}\right)\right]$$

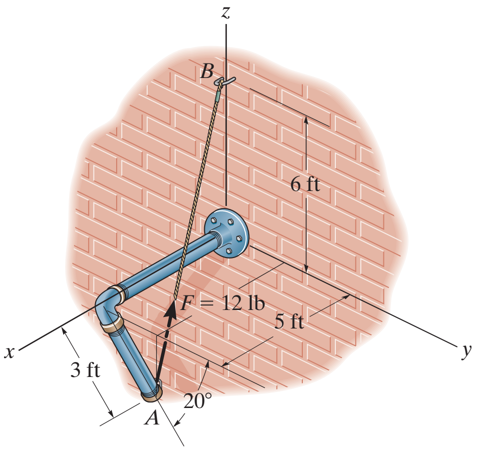
$$-360\mathbf{k} = -\frac{24}{7}\mathbf{k}$$

Thus,

$$360 = \frac{24}{7}F \qquad F = 105 \text{ lb} \qquad \textbf{Ans.}$$



2–107. The pipe is supported at its end by a cord *AB*. If the cord exerts a force of $F = 12$ lb on the pipe at *A*, express this force as a Cartesian vector.



Unit Vector : The coordinates of point *A* are

$$A(5,\ 3\cos 20°,\ -3\sin 20°)\ \text{ft} = A(5.00,\ 2.819,\ -1.026)\ \text{ft}$$

Then

$$\mathbf{r}_{AB} = \{(0-5.00)\mathbf{i} + (0-2.819)\mathbf{j} + [6-(-1.026)]\mathbf{k}\}\ \text{ft}$$
$$= \{-5.00\mathbf{i} - 2.819\mathbf{j} + 7.026\mathbf{k}\}\ \text{ft}$$
$$r_{AB} = \sqrt{(-5.00)^2 + (-2.819)^2 + 7.026^2} = 9.073\ \text{ft}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{-5.00\mathbf{i} - 2.819\mathbf{j} + 7.026\mathbf{k}}{9.073}$$
$$= -0.5511\mathbf{i} - 0.3107\mathbf{j} + 0.7744\mathbf{k}$$

Force Vector :

$$\mathbf{F} = F\mathbf{u}_{AB} = 12\{-0.5511\mathbf{i} - 0.3107\mathbf{j} + 0.7744\mathbf{k}\}\ \text{lb}$$
$$= \{-6.61\mathbf{i} - 3.73\mathbf{j} + 9.29\mathbf{k}\}\ \text{lb} \qquad \textbf{Ans}$$

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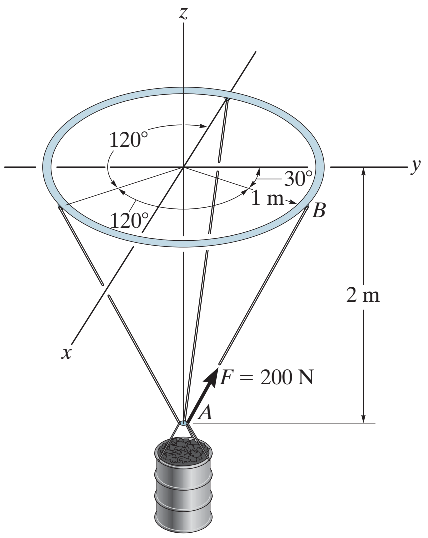
*__2–108.__ The load at *A* creates a force of 200 N in wire *AB*. Express this force as a Cartesian vector, acting on *A* and directed towards *B*.



$$\mathbf{r}_{AB} = (1\sin 30° - 0)\mathbf{i} + (1\cos 30° - 0)\mathbf{j} + (2-0)\mathbf{k}$$

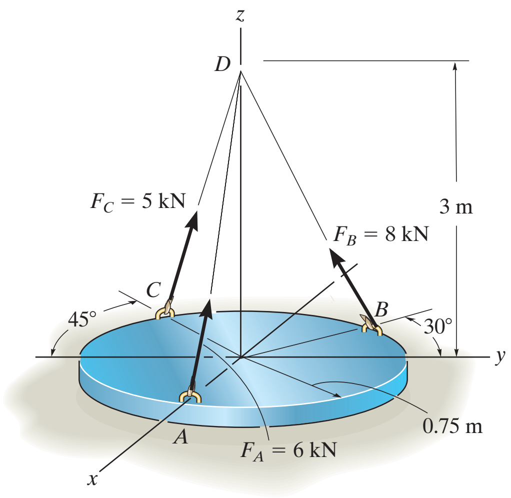
$$= \{0.5\,\mathbf{i} + 0.866\,\mathbf{j} + 2\mathbf{k}\}\text{m}$$

$$r_{AB} = \sqrt{(0.5)^2 + (0.866)^2 + (2)^2} = 2.236\ \text{m}$$

$$\mathbf{u}_{AB} = \left(\frac{\mathbf{r}_{AB}}{r_{AB}}\right) = 0.2236\mathbf{i} + 0.3873\,\mathbf{j} + 0.8944\,\mathbf{k}$$

$$\mathbf{F} = 200\,\mathbf{u}_{AB} = \{44.7\,\mathbf{i} + 77.5\,\mathbf{j} + 179\,\mathbf{k}\}\ \text{N} \quad . \quad \textbf{A}$$

•2–109. The cylindrical plate is subjected to the three cable forces which are concurrent at point *D*. Express each force which the cables exert on the plate as a Cartesian vector, and determine the magnitude and coordinate direction angles of the resultant force.



$$\mathbf{r}_A = (0-0.75)\mathbf{i} + (0-0)\mathbf{j} + (3-0)\mathbf{k} = \{-0.75\,\mathbf{i} + 0\,\mathbf{j} + 3\,\mathbf{k}\}\text{ m}$$

$$r_A = \sqrt{(-0.75)^2 + 0^2 + 3^2} = 3.0923\text{ m}$$

$$\mathbf{F}_A = F_A\left(\frac{\mathbf{r}_A}{r_A}\right) = 6\left(\frac{-0.75\,\mathbf{i} + 3\,\mathbf{k}}{3.0923}\right)$$

$$= \{-1.4552\,\mathbf{i} + 5.8209\,\mathbf{k}\}\text{ kN}$$

$$= \{-1.46\,\mathbf{i} + 5.82\,\mathbf{k}\}\text{ kN} \qquad \textbf{Ans}$$

$$\mathbf{r}_C = \left[0-(-0.75\sin 45^\circ)\right]\mathbf{i} + \left[0-(-0.75\cos 45^\circ)\right]\mathbf{j} + (3-0)\mathbf{k}$$

$$= \{0.5303\,\mathbf{i} + 0.5303\,\mathbf{j} + 3\,\mathbf{k}\}\text{ m}$$

$$r_C = \sqrt{(0.5303)^2 + (0.5303)^2 + 3^2} = 3.0923\text{ m}$$

$$\mathbf{F}_C = F_C\left(\frac{\mathbf{r}_C}{r_C}\right) = 5\left(\frac{0.5303\,\mathbf{i} + 0.5303\,\mathbf{j} + \hat{3}\,\mathbf{k}}{3.0923}\right)$$

$$= \{0.8575\,\mathbf{i} + 0.8575\,\mathbf{j} + 4.8507\,\mathbf{k}\}\text{ kN}$$

$$= \{0.857\,\mathbf{i} + 0.857\,\mathbf{j} + 4.85\,\mathbf{k}\}\text{ kN} \qquad \textbf{Ans}$$

$$\mathbf{r}_B = \left[0-(-0.75\sin 30^\circ)\right]\mathbf{i} + (0-0.75\cos 30^\circ)\mathbf{j} + (3-0)\mathbf{k}$$

$$= \{0.375\,\mathbf{i} - 0.6495\,\mathbf{j} + 3\,\mathbf{k}\}\text{ m}$$

$$r_B = \sqrt{(0.375)^2 + (-0.6495)^2 + 3^2} = 3.0923\text{ m}$$

$$\mathbf{F}_B = F_B\left(\frac{\mathbf{r}_B}{r_B}\right) = 8\left(\frac{0.375\,\mathbf{i} - 0.6495\,\mathbf{j} + 3\,\mathbf{k}}{3.0923}\right)$$

$$= \{0.9701\mathbf{i} - 1.6803\mathbf{j} + 7.7611\mathbf{k}\}\text{ kN}$$

$$= \{0.970\mathbf{i} - 1.68\mathbf{j} + 7.76\mathbf{k}\}\text{ kN} \qquad \textbf{Ans}$$

$$\mathbf{F}_R = \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C$$

$$= \{-1.4552\,\mathbf{i} + 5.8209\,\mathbf{k}\} + \{0.9701\,\mathbf{i} - 1.6803\,\mathbf{j} + 7.7611\mathbf{k}\}$$

$$+ \{0.8575\,\mathbf{i} + 0.8575\,\mathbf{j} + 4.8507\,\mathbf{k}\}$$

$$= \{0.3724\,\mathbf{i} - 0.8228\,\mathbf{j} + 18.4327\,\mathbf{k}\}\text{ kN}$$

$$F_R = \sqrt{(0.3724)^2 + (-0.8228)^2 + (18.4327)^2}$$

$$= 18.4548\text{ kN} = 18.5\text{ kN} \qquad \textbf{Ans}$$

$$\mathbf{u}_R = \frac{\mathbf{F}_R}{F_R} = \frac{0.3724\mathbf{i} - 0.8228\mathbf{j} + 18.4327\mathbf{k}}{18.4548}$$

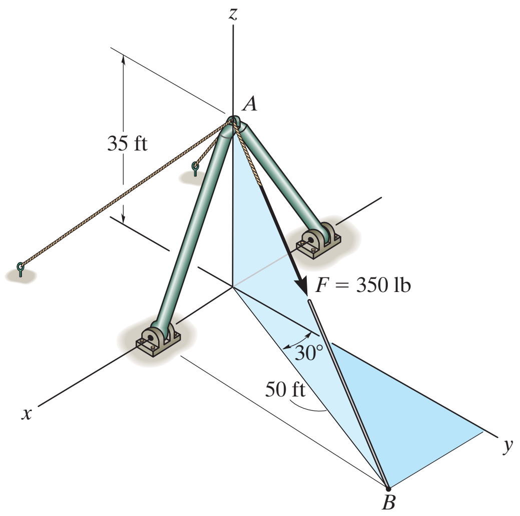
$$= 0.02018\mathbf{i} - 0.04459\mathbf{j} + 0.9988\mathbf{k}$$

$\cos\alpha = 0.02018$ $\qquad \alpha = 88.8^\circ$ **Ans**

$\cos\beta = -0.04458$ $\qquad \beta = 92.6^\circ$ **Ans**

$\cos\gamma = 0.9988$ $\qquad \gamma = 2.81^\circ$ **Ans**

2–110. The cable attached to the shear-leg derrick exerts a force on the derrick of $F = 350$ lb. Express this force as a Cartesian vector.



Unit Vector : The coordinates of point B are

$$B(50\sin 30°,\ 50\cos 30°,\ 0)\ \text{ft} = B(25.0,\ 43.301,\ 0)\ \text{ft}$$

Then

$$\mathbf{r}_{AB} = \{(25.0-0)\mathbf{i}+(43.301-0)\mathbf{j}+(0-35)\mathbf{k}\}\ \text{ft}$$
$$= \{25.0\mathbf{i}+43.301\mathbf{j}-35.0\mathbf{k}\}\ \text{ft}$$
$$r_{AB} = \sqrt{25.0^2+43.301^2+(-35.0)^2} = 61.033\ \text{ft}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{25.0\mathbf{i}+43.301\mathbf{j}-35.0\mathbf{k}}{61.033}$$
$$= 0.4096\mathbf{i}+0.7094\mathbf{j}-0.5735\mathbf{k}$$

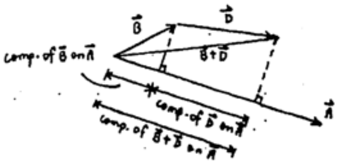
Force Vector :

$$\mathbf{F} = F\mathbf{u}_{AB} = 350\{0.4096\mathbf{i}+0.7094\mathbf{j}-0.5735\mathbf{k}\}\ \text{lb}$$
$$= \{143\mathbf{i}+248\mathbf{j}-201\mathbf{k}\}\ \text{lb} \qquad \textbf{Ans}$$

2–111. Given the three vectors A, B, and D, show that $\mathbf{A}\cdot(\mathbf{B}+\mathbf{D}) = (\mathbf{A}\cdot\mathbf{B}) + (\mathbf{A}\cdot\mathbf{D})$.

Since the component of (**B** + **D**) is equal to the sum of the components of **B** and **D**, then

$$\mathbf{A}\cdot(\mathbf{B}+\mathbf{D}) = \mathbf{A}\cdot\mathbf{B} + \mathbf{A}\cdot\mathbf{D} \qquad \textbf{(QED)}$$


$\vec{B}$
$\vec{D}$
$\vec{B}+\vec{D}$
comp. of $\vec{B}$ on $\vec{A}$
comp. of $\vec{D}$ on $\vec{A}$
comp. of $\vec{B}+\vec{D}$ on $\vec{A}$
$\vec{A}$

Also,

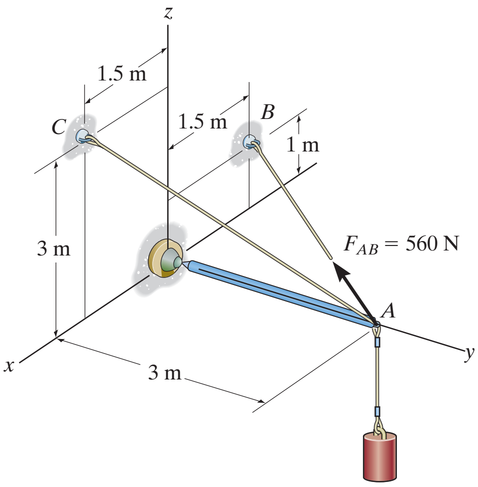
$$\mathbf{A}\cdot(\mathbf{B}+\mathbf{D}) = (A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k})\cdot[(B_x + D_x)\mathbf{i} + (B_y + D_y)\mathbf{j} + (B_z + D_z)\mathbf{k}]$$

$$= A_x(B_x + D_x) + A_y(B_y + D_y) + A_z(B_z + D_z)$$

$$= (A_xB_x + A_yB_y + A_zB_z) + (A_xD_x + A_yD_y + A_zD_z)$$

$$= (\mathbf{A}\cdot\mathbf{B}) + (\mathbf{A}\cdot\mathbf{D}) \qquad \textbf{(QED)}$$

***2–112.** Determine the projected component of the force $F_{AB} = 560$ N acting along cable AC. Express the result as a Cartesian vector.



Force Vectors: The unit vectors $\mathbf{u}_{AB}$ and $\mathbf{u}_{AC}$ must be determined first. From Fig. *a*,

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{(-1.5-0)\mathbf{i}+(0-3)\mathbf{j}+(1-0)\mathbf{k}}{\sqrt{(-1.5-0)^2+(0-3)^2+(1-0)^2}} = -\frac{3}{7}\mathbf{i}-\frac{6}{7}\mathbf{j}+\frac{2}{7}\mathbf{k}$$

$$\mathbf{u}_{AC} = \frac{\mathbf{r}_{AC}}{r_{AC}} = \frac{(1.5-0)\mathbf{i}+(0-3)\mathbf{j}+(3-0)\mathbf{k}}{\sqrt{(1.5-0)^2+(0-3)^2+(3-0)^2}} = \frac{1}{3}\mathbf{i}-\frac{2}{3}\mathbf{j}+\frac{2}{3}\mathbf{k}$$

Thus, the force vector $\mathbf{F}_{AB}$ is given by

$$\mathbf{F}_{AB} = F_{AB}\,\mathbf{u}_{AB} = 560\left(-\frac{3}{7}\mathbf{i}-\frac{6}{7}\mathbf{j}+\frac{2}{7}\mathbf{k}\right) = [-240\mathbf{i}-480\mathbf{j}+160\mathbf{k}]\text{N}$$

Vector Dot Product: The magnitude of the projected component of $\mathbf{F}_{AB}$ is

$$(F_{AB})_{AC} = \mathbf{F}_{AB}\cdot\mathbf{u}_{AC} = (-240\mathbf{i}-480\mathbf{j}+160\mathbf{k})\cdot\left(\frac{1}{3}\mathbf{i}-\frac{2}{3}\mathbf{j}+\frac{2}{3}\mathbf{k}\right)$$

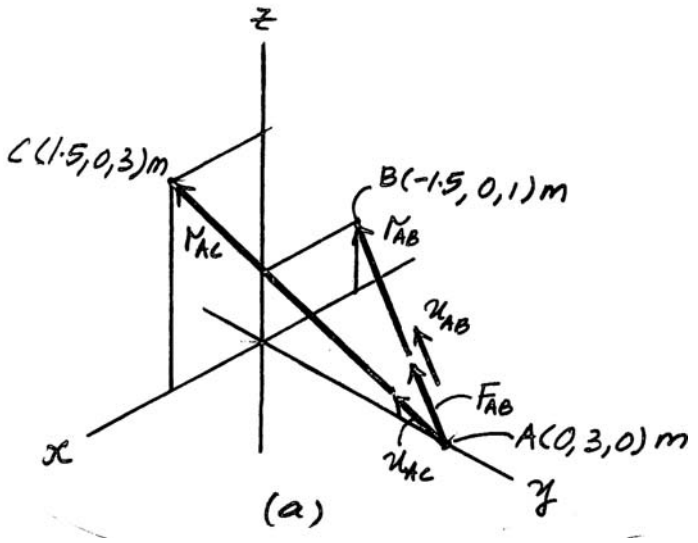
$$= (-240)\left(\frac{1}{3}\right)+(-480)\left(-\frac{2}{3}\right)+160\left(\frac{2}{3}\right)$$

$$= 346.67\text{ N}$$

Thus, $(\mathbf{F}_{AB})_{AC}$ expressed in Cartesian vector form is

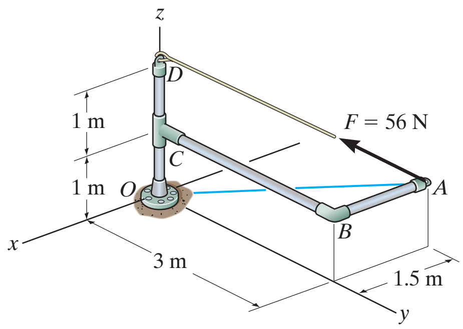
$$(\mathbf{F}_{AB})_{AC} = (F_{AB})_{AC}\,\mathbf{u}_{AC} = 346.67\left(\frac{1}{3}\mathbf{i}-\frac{2}{3}\mathbf{j}+\frac{2}{3}\mathbf{k}\right)$$

$$= [116\mathbf{i}-231\mathbf{j}+231\mathbf{k}]\text{N} \qquad \textbf{Ans.}$$



(a)

•2–113. Determine the magnitudes of the components of force $F = 56$ N acting along and perpendicular to line *AO*.



Unit Vectors: The unit vectors $\mathbf{u}_{AD}$ and $\mathbf{u}_{AO}$ must be determined first. From Fig. *a*,

$$\mathbf{u}_{AD} = \frac{\mathbf{r}_{AD}}{r_{AD}} = \frac{[0-(-1.5)]\mathbf{i} + (0-3)\mathbf{j} + (2-1)\mathbf{k}}{\sqrt{[0-(-1.5)]^2 + (0-3)^2 + (2-1)^2}} = \frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}$$

$$\mathbf{u}_{AO} = \frac{\mathbf{r}_{AO}}{r_{AO}} = \frac{[0-(-1.5)]\mathbf{i} + (0-3)\mathbf{j} + (0-1)\mathbf{k}}{\sqrt{[0-(-1.5)]^2 + (0-3)^2 + (0-1)^2}} = \frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} - \frac{2}{7}\mathbf{k}$$

Thus, the force vector **F** is given by

$$\mathbf{F} = F\mathbf{u}_{AD} = 56\left(\frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}\right) = [24\mathbf{i} - 48\mathbf{j} + 16\mathbf{k}]\text{N}$$

Vector Dot Product: The magnitude of the projected component of **F** parallel to line *AO* is

$$(\mathbf{F}_{AO})_{\text{paral}} = \mathbf{F}\cdot\mathbf{u}_{AO} = (24\mathbf{i} - 48\mathbf{j} + 16\mathbf{k})\cdot\left(\frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} - \frac{2}{7}\mathbf{k}\right)$$

$$= (24)\left(\frac{3}{7}\right) + (-48)\left(-\frac{6}{7}\right) + (16)\left(-\frac{2}{7}\right)$$

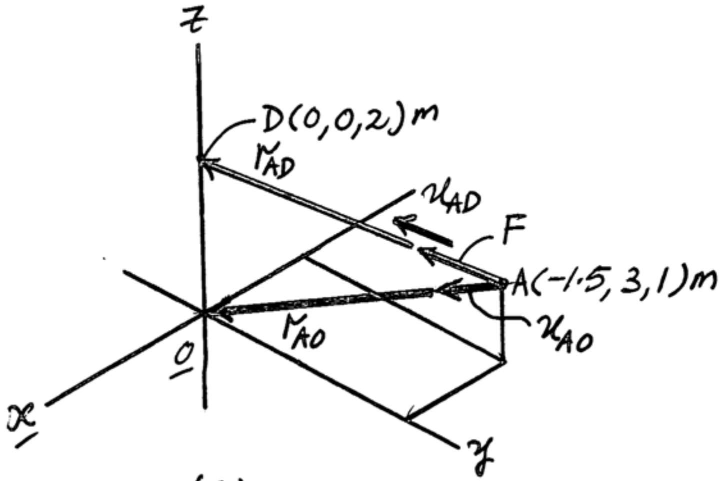
$$= 46.86\text{ N} = 46.9\text{ N}$$ **Ans.**

The component of **F** perpendicular to line *AO* is

$$(\mathbf{F}_{AO})_{\text{per}} = \sqrt{F^2 - (F_{AO})_{\text{paral}}}$$

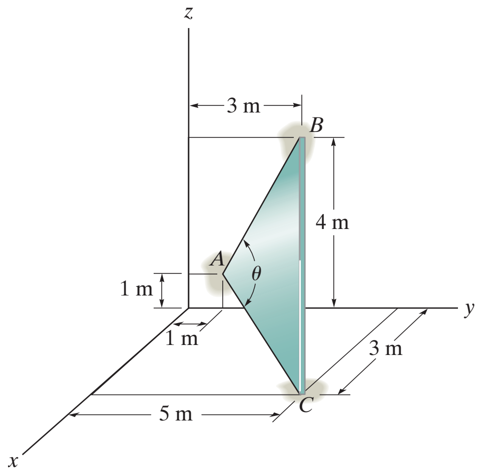
$$= \sqrt{56^2 - 46.86^2}$$

$$= 30.7\text{N}$$ **Ans.**



(a)

2–114. Determine the length of side *BC* of the triangular plate. Solve the problem by finding the magnitude of $\mathbf{r}_{BC}$; then check the result by first finding θ, r_{AB}, and r_{AC} and then using the cosine law.



$\mathbf{r}_{BC} = \{3\,\mathbf{i} + 2\,\mathbf{j} - 4\,\mathbf{k}\}$ m

$r_{BC} = \sqrt{(3)^2 + (2)^2 + (-4)^2} = 5.39$ m **Ans**

Also,

$\mathbf{r}_{AC} = \{3\,\mathbf{i} + 4\,\mathbf{j} - 1\,\mathbf{k}\}$ m

$r_{AC} = \sqrt{(3)^2 + (4)^2 + (-1)^2} = 5.0990$ m

$\mathbf{r}_{AB} = \{2\,\mathbf{j} + 3\,\mathbf{k}\}$ m

$r_{AB} = \sqrt{(2)^2 + (3)^2} = 3.6056$ m

$\mathbf{r}_{AC} \cdot \mathbf{r}_{AB} = 0 + 4(2) + (-1)(3) = 5$

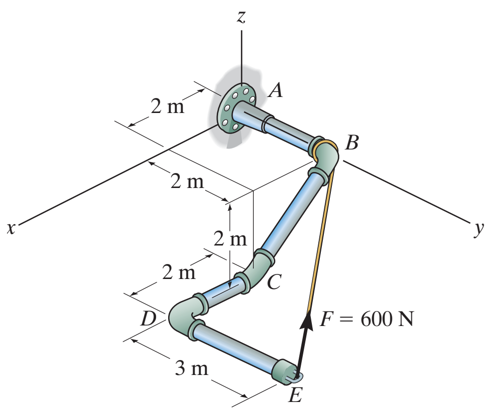
$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_{AC} \cdot \mathbf{r}_{AB}}{r_{AC}r_{AB}}\right) = \cos^{-1}\frac{5}{(5.0990)(3.6056)}$$

$\theta = 74.219°$

$$r_{BC} = \sqrt{(5.0990)^2 + (3.6056)^2 - 2(5.0990)(3.6056)\cos 74.219°}$$

$r_{BC} = 5.39$ m **Ans**

2–115. Determine the magnitudes of the components of $F = 600$ N acting along and perpendicular to segment DE of the pipe assembly.



Unit Vectors: The unit vectors $\mathbf{u}_{EB}$ and $\mathbf{u}_{ED}$ must be determined first. From Fig. *a*,

$$\mathbf{u}_{EB} = \frac{\mathbf{r}_{EB}}{r_{EB}} = \frac{(0-4)\mathbf{i} + (2-5)\mathbf{j} + [0-(-2)]\mathbf{k}}{\sqrt{(0-4)^2 + (2-5)^2 + [0-(-2)]^2}} = -0.7428\mathbf{i} - 0.5571\mathbf{j} + 0.3714\mathbf{k}$$

$$\mathbf{u}_{ED} = -\mathbf{j}$$

Thus, the force vector **F** is given by

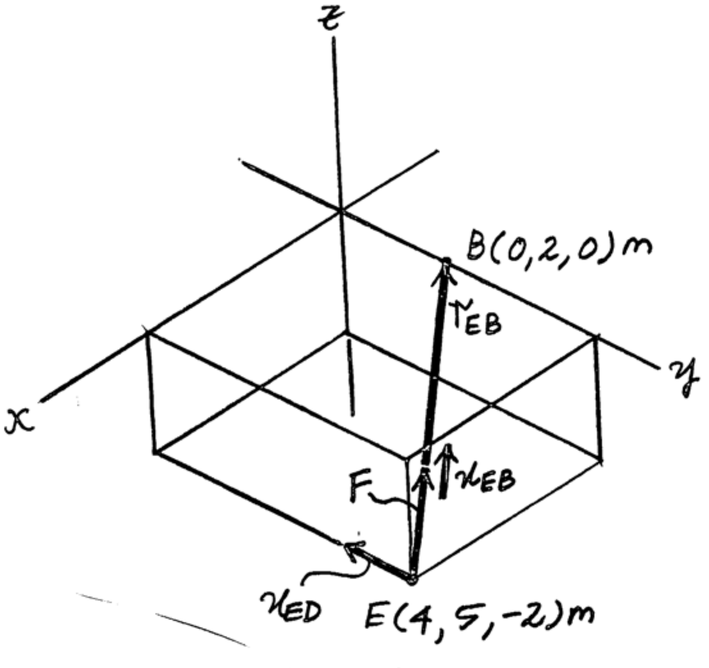
$$\mathbf{F} = F\mathbf{u}_{EB} = 600(-0.7428\mathbf{i} - 0.5571\mathbf{j} + 0.3714\mathbf{k}) = [-445.66\mathbf{i} - 334.25\mathbf{j} + 222.83\mathbf{k}]\ \text{N}$$

Vector Dot Product: The magnitude of the component of **F** parallel to segment *DE* of the pipe assembly is

$$\begin{aligned}(F_{ED})_{\text{paral}} = \mathbf{F}\cdot\mathbf{u}_{ED} &= (-445.66\mathbf{i} - 334.25\mathbf{j} + 222.83\mathbf{k})\cdot(-\mathbf{j})\\ &= (-445.66)(0) + (-334.25)(-1) + (222.83)(0)\\ &= 334.25 = 334\ \text{N}\end{aligned}$$ **Ans.**

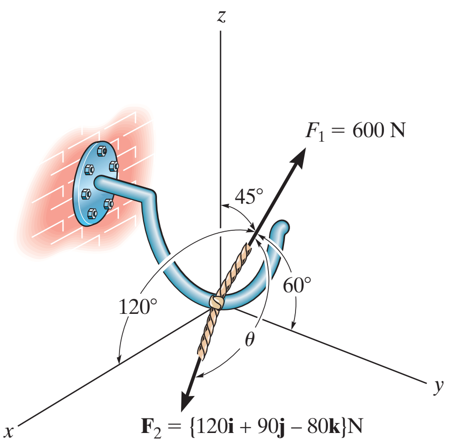
The component of **F** perpendicular to segment *DE* of the pipe assembly is

$$(F_{ED})_{\text{per}} = \sqrt{F^2 - (F_{ED})_{\text{paral}}{}^2} = \sqrt{600^2 - 334.25^2} = 498\ \text{N}$$ **Ans.**



(a)

***2–116.** Two forces act on the hook. Determine the angle θ between them. Also, what are the projections of $\mathbf{F}_1$ and $\mathbf{F}_2$ along the y axis?


z
$F_1 = 600$ N
45°
60°
120°
θ
y
x
$\mathbf{F}_2 = \{120\mathbf{i} + 90\mathbf{j} - 80\mathbf{k}\}$N

$$\mathbf{F}_1 = 600\cos 120^\circ\,\mathbf{i} + 600\cos 60^\circ\,\mathbf{j} + 600\cos 45^\circ\,\mathbf{k}$$

$$= -300\,\mathbf{i} + 300\,\mathbf{j} + 424.3\,\mathbf{k}; \quad F_1 = 600\text{ N}$$

$$\mathbf{F}_2 = 120\,\mathbf{i} + 90\,\mathbf{j} - 80\,\mathbf{k}; \quad F_2 = 170\text{ N}$$

$$\mathbf{F}_1 \cdot \mathbf{F}_2 = (-300)(120) + (300)(90) + (424.3)(-80) = -42\,944$$

$$\theta = \cos^{-1}\left(\frac{-42\,944}{(170)(600)}\right) = 115^\circ \qquad \textbf{Ans}$$

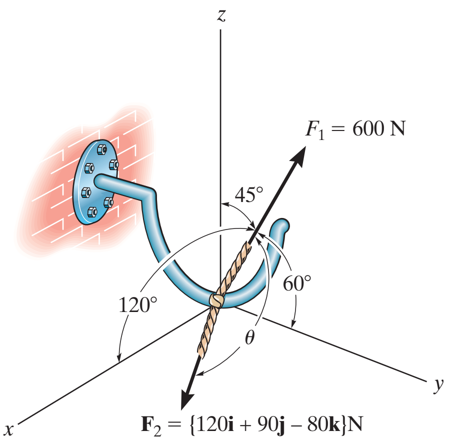
$$\mathbf{u} = \mathbf{j}$$

So,

$$F_{1y} = \mathbf{F}_1 \cdot \mathbf{j} = (300)(1) = 300\text{ N} \qquad \textbf{Ans}$$

$$F_{2y} = \mathbf{F}_2 \cdot \mathbf{j} = (90)(1) = 90\text{ N} \qquad \textbf{Ans}$$

•2–117. Two forces act on the hook. Determine the magnitude of the projection of $\mathbf{F}_2$ along $\mathbf{F}_1$.

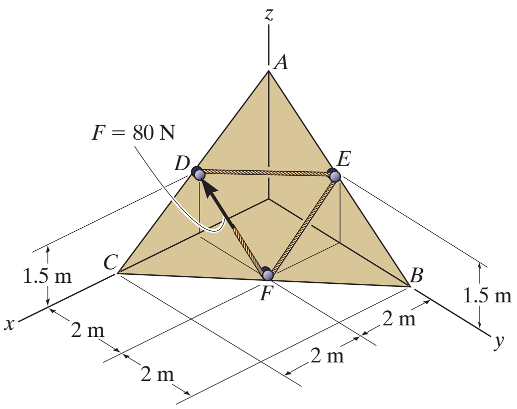

z
$F_1 = 600$ N
45°
60°
120°
θ
y
x
$\mathbf{F}_2 = \{120\mathbf{i} + 90\mathbf{j} - 80\mathbf{k}\}$N

$$\mathbf{u}_1 = \cos 120^\circ\,\mathbf{i} + \cos 60^\circ\,\mathbf{j} + \cos 45^\circ\,\mathbf{k}$$

$$\text{Proj } F_2 = \mathbf{F}_2 \cdot \mathbf{u}_1 = (120)(\cos 120^\circ) + (90)(\cos 60^\circ) + (-80)(\cos 45^\circ)$$

$$|\text{Proj } F_2| = 71.6\text{ N} \qquad \textbf{Ans}$$

2–118. Determine the projection of force $F = 80$ N along line BC. Express the result as a Cartesian vector.



Unit Vectors: The unit vectors $\mathbf{u}_{FD}$ and $\mathbf{u}_{FC}$ must be determined first. From Fig. *a*,

$$\mathbf{u}_{FD} = \frac{\mathbf{r}_{FD}}{r_{FD}} = \frac{(2-2)\mathbf{i}+(0-2)\mathbf{j}+(1.5-0)\mathbf{k}}{\sqrt{(2-2)^2+(0-2)^2+(1.5-0)^2}} = -\frac{4}{5}\mathbf{j}+\frac{3}{5}\mathbf{k}$$

$$\mathbf{u}_{FC} = \frac{\mathbf{r}_{FC}}{r_{FC}} = \frac{(4-2)\mathbf{i}+(0-2)\mathbf{j}+(0-0)\mathbf{k}}{\sqrt{(4-2)^2+(0-2)^2+(0-0)^2}} = 0.7071\mathbf{i}-0.7071\mathbf{j}$$

Thus, the force vector **F** is given by

$$\mathbf{F} = F\mathbf{u}_{FD} = 80\left(-\frac{4}{5}\mathbf{j}+\frac{3}{5}\mathbf{k}\right) = [-64\mathbf{j}+48\mathbf{k}]\text{ N}$$

Vector Dot Product: The magnitude of the projected component of **F** along line BC is

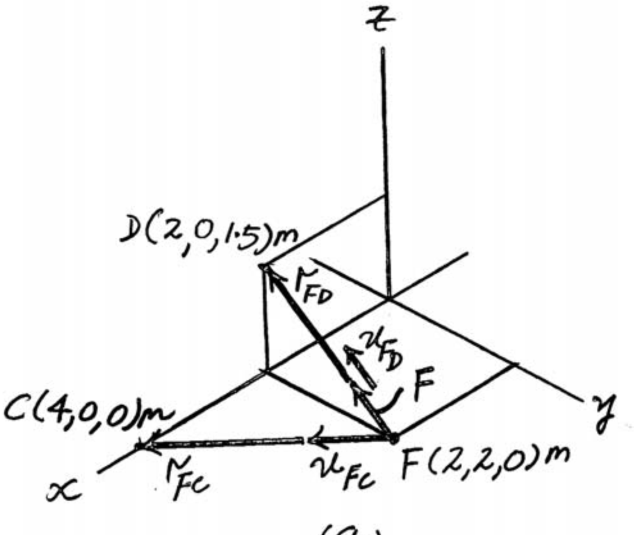
$$\begin{aligned} F_{BC} &= \mathbf{F}\cdot\mathbf{u}_{FC} = (-64\mathbf{j}+48\mathbf{k})\cdot(0.7071\mathbf{i}-0.7071\mathbf{j}) \\ &= (0)(0.7071)+(-64)(-0.7071)+48(0) \\ &= 45.25 = 45.2\text{ N} \end{aligned}$$

Ans.

The component of $\mathbf{F}_{BC}$ can be expressed in Cartesian vector form as

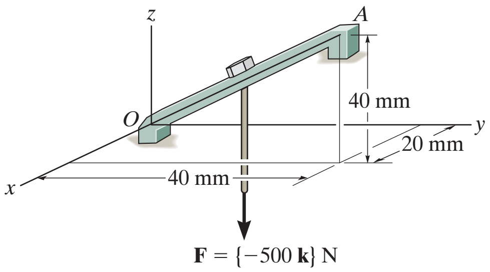
$$\begin{aligned} \mathbf{F}_{BC} &= F_{BC}(\mathbf{u}_{FC}) = 45.25(0.7071\mathbf{i}-0.7071\mathbf{j}) \\ &= \{32\mathbf{i}-32\mathbf{j}\}\text{ N} \end{aligned}$$

Ans.



(a)

2–119. The clamp is used on a jig. If the vertical force acting on the bolt is $\mathbf{F} = \{-500\mathbf{k}\}$ N, determine the magnitudes of its components $\mathbf{F}_1$ and $\mathbf{F}_2$ which act along the *OA* axis and perpendicular to it.



Unit Vector : The unit vector along *OA* axis is

$$\mathbf{u}_{AO} = \frac{(0-20)\mathbf{i} + (0-40)\mathbf{j} + (0-40)\mathbf{k}}{\sqrt{(0-20)^2 + (0-40)^2 + (0-40)^2}} = -\frac{1}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} - \frac{2}{3}\mathbf{k}$$

Projected Component of F Along OA Axis :

$$F_1 = \mathbf{F} \cdot \mathbf{u}_{AO} = (-500\mathbf{k}) \cdot \left(-\frac{1}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} - \frac{2}{3}\mathbf{k}\right)$$

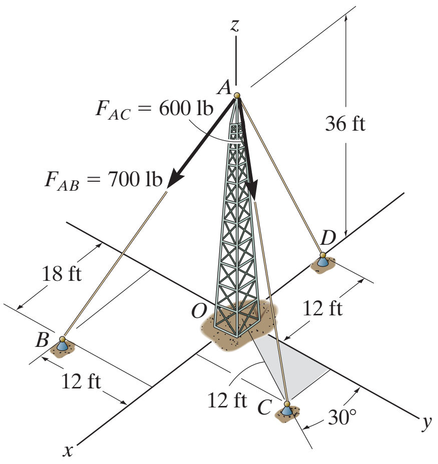
$$= (0)\left(-\frac{1}{3}\right) + (0)\left(-\frac{2}{3}\right) + (-500)\left(-\frac{2}{3}\right)$$

$$= 333.33 \text{ N} = 333 \text{ N} \qquad \textbf{Ans}$$

Component of F Perpendicular to OA Axis : Since the magnitude of force **F** is $F = 500$ N so that

$$F_2 = \sqrt{F^2 - F_1^2} = \sqrt{500^2 - 333.33^2} = 373 \text{ N} \qquad \textbf{Ans}$$

***2–120.** Determine the magnitude of the projected component of force $\mathbf{F}_{AB}$ acting along the z axis.



Unit Vector: The unit vector $\mathbf{u}_{AB}$ must be determined first. From Fig. *a*,

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{(18-0)\mathbf{i} + (-12-0)\mathbf{j} + (0-36)\mathbf{k}}{\sqrt{(18-0)^2 + (-12-0)^2 + (0-36)^2}} = \frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

Thus, the force vector $\mathbf{F}_{AB}$ is given by

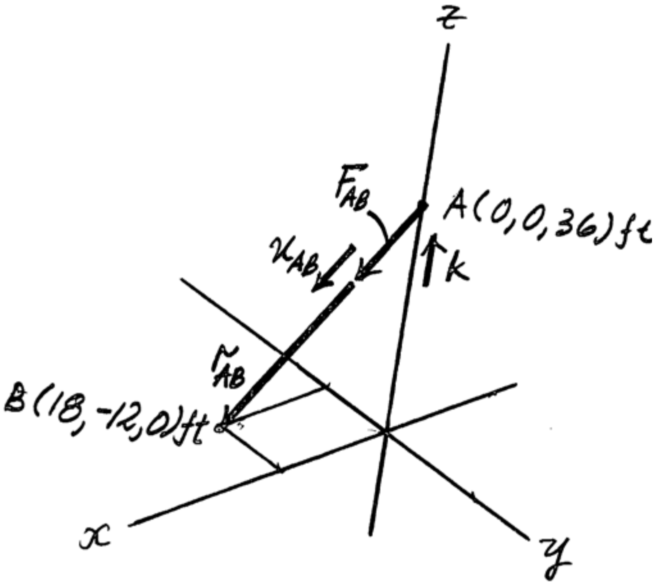
$$\mathbf{F}_{AB} = F_{AB}\,\mathbf{u}_{AB} = 700\left(\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = \{300\mathbf{i} - 200\mathbf{j} - 600\mathbf{k}\}\text{ lb}$$

Vector Dot Product: The projected component of $\mathbf{F}_{AB}$ along the z axis is

$$(F_{AB})_z = \mathbf{F}_{AB}\cdot\mathbf{k} = (300\mathbf{i} - 200\mathbf{j} - 600\mathbf{k})\cdot\mathbf{k}$$
$$= -600\text{ lb}$$

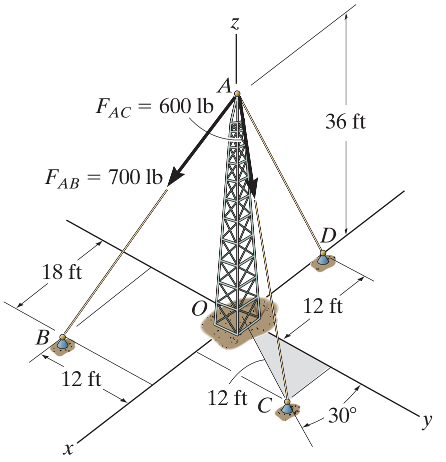
The negative sign indicates that $(\mathbf{F}_{AB})_z$ is directed towards the negative z axis. Thus

$(F_{AB})_z = 600$ lb **Ans.**



(a)

•2–121. Determine the magnitude of the projected component of force $\mathbf{F}_{AC}$ acting along the z axis.



Unit Vector: The unit vector $\mathbf{u}_{AC}$ must be determined first. From Fig. *a*,

$$\mathbf{u}_{AC} = \frac{\mathbf{r}_{AC}}{r_{AC}} = \frac{(12\sin 30° - 0)\mathbf{i} + (12\cos 30° - 0)\mathbf{j} + (0 - 36)\mathbf{k}}{\sqrt{(12\sin 30° - 0)^2 + (12\cos 30° - 0)^2 + (0 - 36)^2}} = 0.1581\mathbf{i} + 0.2739\mathbf{j} - 0.9487\mathbf{k}$$

Thus, the force vector $\mathbf{F}_{AC}$ is given by

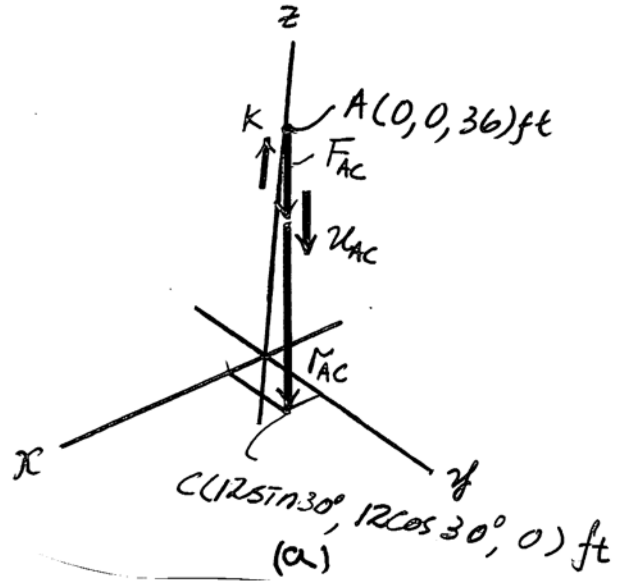
$$\mathbf{F}_{AC} = F_{AC}\mathbf{u}_{AC} = 600(0.1581\mathbf{i} + 0.2739\mathbf{j} - 0.9487\mathbf{k}) = \{94.87\mathbf{i} + 164.32\mathbf{j} - 569.21\mathbf{k}\}\text{N}$$

Vector Dot Product: The projected component of $\mathbf{F}_{AC}$ along the z axis is

$$(F_{AC})_z = \mathbf{F}_{AC} \cdot \mathbf{k} = (94.87\mathbf{i} + 164.32\mathbf{j} - 569.21\mathbf{k}) \cdot \mathbf{k}$$
$$= -569 \text{ lb}$$

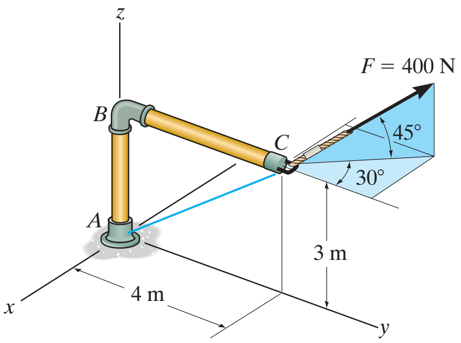
The negative sign indicates that $(\mathbf{F}_{AC})_z$ is directed towards the negative z axis. Thus

$(F_{AC})_z = 569$ lb **Ans.**



(a)

2–122. Determine the projection of force $F = 400$ N acting along line AC of the pipe assembly. Express the result as a Cartesian vector.



Force and unit Vector: The force vector **F** and unit vector $\mathbf{u}_{AC}$ must be determined first. From Fig. (*a*)

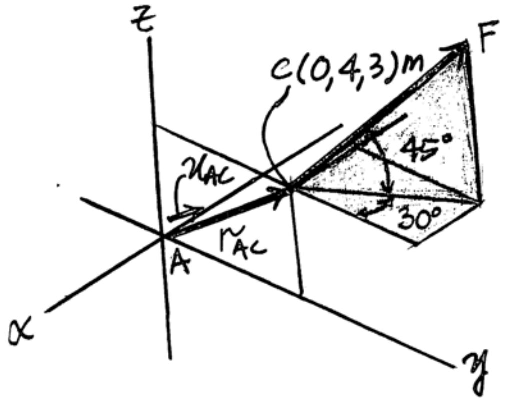
$$\mathbf{F} = 400(-\cos 45^\circ \sin 30^\circ\,\mathbf{i} + \cos 45^\circ \cos 30^\circ\,\mathbf{j} + \sin 45^\circ\,\mathbf{k})$$
$$=\{-141.42\mathbf{i} + 244.95\mathbf{j} + 282.84\mathbf{k}\}$$
$$\mathbf{u}_{AC} = \frac{\mathbf{r}_{AC}}{r_{AC}} = \frac{(0-0)\mathbf{i} + (4-0)\mathbf{j} + (3-0)\mathbf{k}}{\sqrt{(0-0)^2 + (4-0)^2 + (3-0)^2}} = \frac{4}{5}\mathbf{j} + \frac{3}{5}\mathbf{k}$$

Vector Dot Product: The magnitude of the projected component of **F** along line AC is

$$F_{AC} = \mathbf{F}\cdot\mathbf{u}_{AC} = (-141.42\mathbf{i} + 244.95\mathbf{j} + 282.84\mathbf{k})\cdot\left(\frac{4}{5}\mathbf{j} + \frac{3}{5}\mathbf{k}\right)$$
$$=(-141.42)(0) + 244.95\left(\frac{4}{5}\right) + 282.84\left(\frac{3}{5}\right)$$
$$= 365.66\ \text{lb}$$ **Ans.**

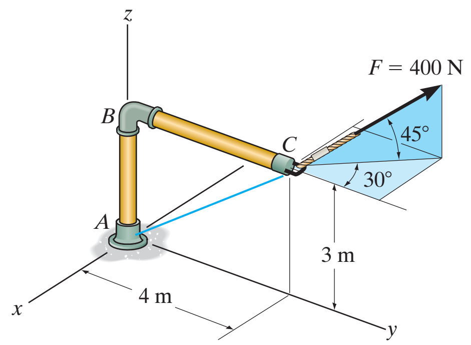
Thus, $\mathbf{F}_{AC}$ written in Cartesian vector form is

$$\mathbf{F}_{AC} = F_{AC}\,\mathbf{u}_{AC} = 365.66\left(\frac{4}{5}\mathbf{j} + \frac{3}{5}\mathbf{k}\right) = \{293\mathbf{j} + 219\mathbf{k}\}\ \text{lb}$$ **Ans.**



(a)

2–123. Determine the magnitudes of the components of force $F = 400$ N acting parallel and perpendicular to segment BC of the pipe assembly.



Force Vector: The force vector **F** must be determined first. From Fig. *a*,

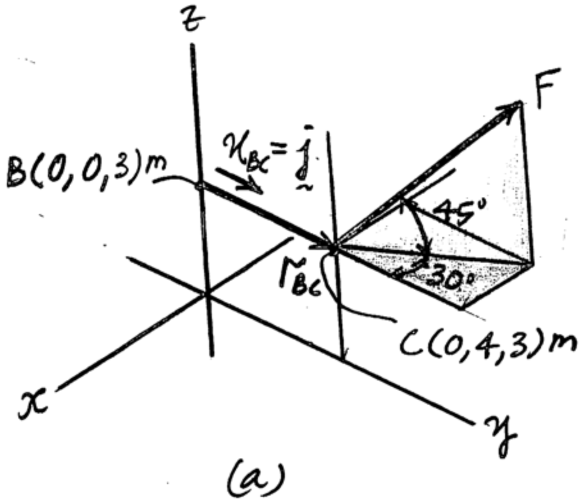
$$\mathbf{F} = 400(-\cos 45° \sin 30° \mathbf{i} + \cos 45° \cos 30° \mathbf{j} + \sin 45° \mathbf{k})$$
$$= \{-141.42\mathbf{i} + 244.95\mathbf{j} + 282.84\mathbf{k}\} \text{ N}$$

Vector Dot Product: By inspecting Fig. (*a*) we notice that $u_{BC} = \mathbf{j}$. Thus, the magnitude of the component of **F** parallel to segment BC of the pipe assembly is

$$(F_{BC})_{\text{paral}} = \mathbf{F} \cdot \mathbf{j} = (-141.42\mathbf{i} + 244.95\mathbf{j} + 282.84\mathbf{k}) \cdot \mathbf{j}$$
$$= -141.42(0) + 244.95(1) + 282.84(0)$$
$$= 244.95 \text{ lb} = 245 \text{ N} \qquad \textbf{Ans.}$$

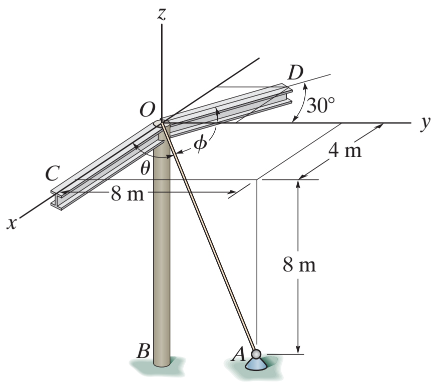
The magnitude of the component of **F** perpendicular to segment BC of the pipe assembly can be determined from

$$(F_{BC})_{\text{per}} = \sqrt{F^2 - (F_{BC})_{\text{paral}}} = \sqrt{400^2 - 244.95^2} = 316 \text{ N} \qquad \textbf{Ans.}$$



(a)

***2–124.** Cable *OA* is used to support column *OB*. Determine the angle θ it makes with beam *OC*.



Unit Vector :

$$\mathbf{u}_{OC} = 1\mathbf{i}$$

$$\mathbf{u}_{OA} = \frac{(4-0)\,\mathbf{i} + (8-0)\,\mathbf{j} + (-8-0)\,\mathbf{k}}{\sqrt{(4-0)^2 + (8-0)^2 + (-8-0)^2}}$$
$$= \frac{1}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} - \frac{2}{3}\mathbf{k}$$

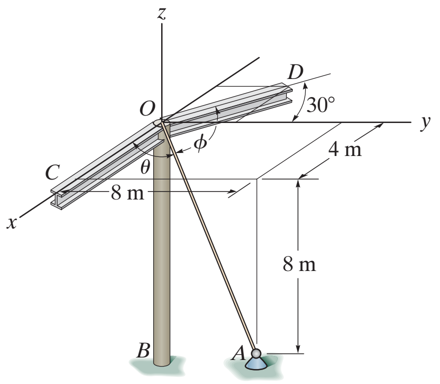
The Angle Between Two Vectors θ :

$$\mathbf{u}_{OC} \cdot \mathbf{u}_{OA} = (1\mathbf{i}) \cdot \left(\frac{1}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} - \frac{2}{3}\mathbf{k}\right) = 1\left(\frac{1}{3}\right) + (0)\left(\frac{2}{3}\right) + 0\left(-\frac{2}{3}\right) = \frac{1}{3}$$

Then,

$$\theta = \cos^{-1}(\mathbf{u}_{OC} \cdot \mathbf{u}_{OA}) = \cos^{-1}\frac{1}{3} = 70.5^\circ \qquad \textbf{Ans}$$

•2–125. Cable *OA* is used to support column *OB*. Determine the angle ϕ it makes with beam *OD*.



Unit Vector :

$$\mathbf{u}_{OD} = -\sin 30^\circ \mathbf{i} + \cos 30^\circ \mathbf{j} = -0.5\mathbf{i} + 0.8660\mathbf{j}$$

$$\mathbf{u}_{OA} = \frac{(4-0)\,\mathbf{i} + (8-0)\,\mathbf{j} + (-8-0)\,\mathbf{k}}{\sqrt{(4-0)^2 + (8-0)^2 + (-8-0)^2}}$$
$$= \frac{1}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} - \frac{2}{3}\mathbf{k}$$

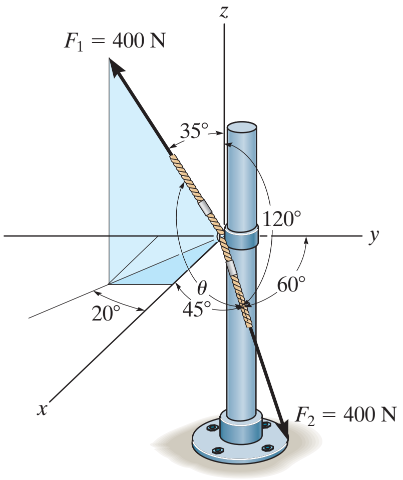
The Angles Between Two Vectors ϕ :

$$\mathbf{u}_{OD} \cdot \mathbf{u}_{OA} = (-0.5\mathbf{i} + 0.8660\mathbf{j}) \cdot \left(\frac{1}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} - \frac{2}{3}\mathbf{k}\right)$$
$$= (-0.5)\left(\frac{1}{3}\right) + (0.8660)\left(\frac{2}{3}\right) + 0\left(-\frac{2}{3}\right)$$
$$= 0.4107$$

Then,

$$\phi = \cos^{-1}(\mathbf{u}_{OD} \cdot \mathbf{u}_{OA}) = \cos^{-1} 0.4107 = 65.8^\circ \qquad \textbf{Ans}$$

2–126. The cables each exert a force of 400 N on the post. Determine the magnitude of the projected component of $\mathbf{F}_1$ along the line of action of $\mathbf{F}_2$.



Force Vector :

$$\mathbf{u}_{F_1} = \sin 35°\cos 20°\mathbf{i} - \sin 35°\sin 20°\mathbf{j} + \cos 35°\mathbf{k}$$
$$= 0.5390\mathbf{i} - 0.1962\mathbf{j} + 0.8192\mathbf{k}$$

$$\mathbf{F}_1 = F_1\mathbf{u}_{F_1} = 400(0.5390\mathbf{i} - 0.1962\mathbf{j} + 0.8192\mathbf{k})\ \text{N}$$
$$= \{215.59\mathbf{i} - 78.47\mathbf{j} + 327.66\mathbf{k}\}\ \text{N}$$

Unit Vector : The unit vector along the line of action of $\mathbf{F}_2$ is

$$\mathbf{u}_{F_2} = \cos 45°\mathbf{i} + \cos 60°\mathbf{j} + \cos 120°\mathbf{k}$$
$$= 0.7071\mathbf{i} + 0.5\mathbf{j} - 0.5\mathbf{k}$$

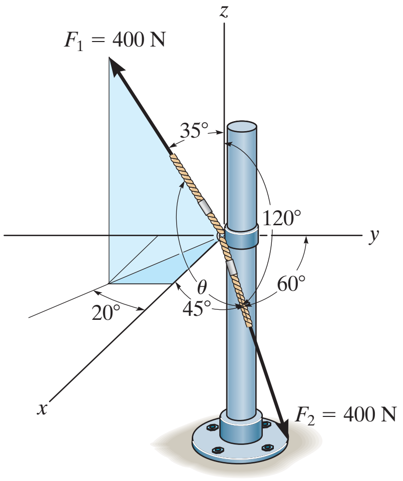
Projected Component of $\mathbf{F}_1$ Along Line of Action of $\mathbf{F}_2$:

$$(F_1)_{F_2} = \mathbf{F}_1 \cdot \mathbf{u}_{F_2} = (215.59\mathbf{i} - 78.47\mathbf{j} + 327.66\mathbf{k}) \cdot (0.7071\mathbf{i} + 0.5\mathbf{j} - 0.5\mathbf{k})$$
$$= (215.59)(0.7071) + (-78.47)(0.5) + (327.66)(-0.5)$$
$$= -50.6\ \text{N}$$

Negative sign indicates that the force component $(\mathbf{F}_1)_{F_2}$ acts in the opposite sense of direction to that of $\mathbf{u}_{F_2}$.

thus the magnitude is $(\mathbf{F}_1)_{F_2} = 50.6$ N **Ans**

2–127. Determine the angle θ between the two cables attached to the post.



Unit Vector :

$$\mathbf{u}_{F_1} = \sin 35°\cos 20°\mathbf{i} - \sin 35°\sin 20°\mathbf{j} + \cos 35°\mathbf{k}$$
$$= 0.5390\mathbf{i} - 0.1962\mathbf{j} + 0.8192\mathbf{k}$$

$$\mathbf{u}_{F_2} = \cos 45°\mathbf{i} + \cos 60°\mathbf{j} + \cos 120°\mathbf{k}$$
$$= 0.7071\mathbf{i} + 0.5\mathbf{j} - 0.5\mathbf{k}$$

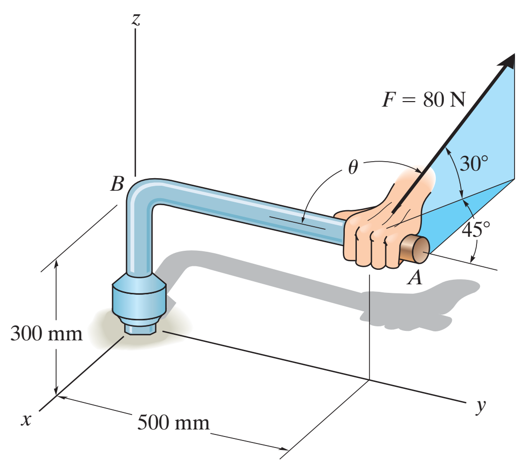
The Angle Between Two Vectors θ : The dot product of two unit vectors must be determined first.

$$\mathbf{u}_{F_1} \cdot \mathbf{u}_{F_2} = (0.5390\mathbf{i} - 0.1962\mathbf{j} + 0.8192\mathbf{k}) \cdot (0.7071\mathbf{i} + 0.5\mathbf{j} - 0.5\mathbf{k})$$
$$= 0.5390(0.7071) + (-0.1962)(0.5) + 0.8192(-0.5)$$
$$= -0.1265$$

Then,

$$\theta = \cos^{-1}\left(\mathbf{u}_{F_1} \cdot \mathbf{u}_{F_2}\right) = \cos^{-1}(-0.1265) = 97.3° \qquad \textbf{Ans}$$

***2–128.** A force of $F = 80\text{ N}$ is applied to the handle of the wrench. Determine the angle θ between the tail of the force and the handle AB.



$$\mathbf{u}_F = -\cos 30° \sin 45° \mathbf{i} + \cos 30° \cos 45° \mathbf{j} + \sin 30° \mathbf{k}$$

$$= -0.6124\,\mathbf{i} + 0.6124\,\mathbf{j} + 0.5\,\mathbf{k}$$

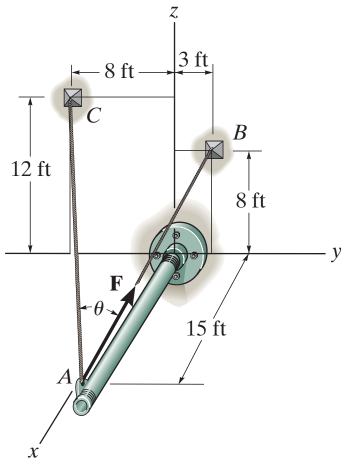
$$\mathbf{u}_{AB} = -\mathbf{j}$$

$$\cos\theta = \mathbf{u}_F \cdot \mathbf{u}_{AB} = (-0.6124\,\mathbf{i} + 0.6124\,\mathbf{j} + 0.5\,\mathbf{k}) \cdot (-\mathbf{j})$$

$$= -0.6124$$

$$\theta = 128° \qquad \textbf{Ans}$$

•2–129. Determine the angle θ between cables AB and AC.



Position Vector :

$$\mathbf{r}_{AB} = \{(0-15)\mathbf{i} + (3-0)\mathbf{j} + (8-0)\mathbf{k}\}\text{ ft}$$
$$= \{-15\mathbf{i} + 3\mathbf{j} + 8\mathbf{k}\}\text{ ft}$$

$$\mathbf{r}_{AC} = \{(0-15)\mathbf{i} + (-8-0)\mathbf{j} + (12-0)\mathbf{k}\}\text{ ft}$$
$$= \{-15\mathbf{i} - 8\mathbf{j} + 12\mathbf{k}\}\text{ ft}$$

The magnitudes of the postion vectors are

$$r_{AB} = \sqrt{(-15)^2 + 3^2 + 8^2} = 17.263\text{ ft}$$
$$r_{AC} = \sqrt{(-15)^2 + (-8)^2 + 12^2} = 20.809\text{ ft}$$

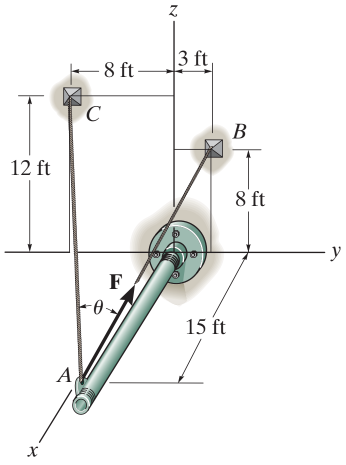
The Angle Between Two Vectors θ:

$$\mathbf{r}_{AB} \cdot \mathbf{r}_{AC} = (-15\mathbf{i} + 3\mathbf{j} + 8\mathbf{k}) \cdot (-15\mathbf{i} - 8\mathbf{j} + 12\mathbf{k})$$
$$= (-15)(-15) + (3)(-8) + 8(12)$$
$$= 297\text{ ft}^2$$

Then,

$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_{AB} \cdot \mathbf{r}_{AC}}{r_{AB} r_{AC}}\right) = \cos^{-1}\left[\frac{297}{17.263(20.809)}\right] = 34.2° \qquad \textbf{Ans}$$

2–130. If **F** has a magnitude of 55 lb, determine the magnitude of its projected components acting along the *x* axis and along cable *AC*.



Force Vector :

$$\mathbf{u}_{AB} = \frac{(0-15)\mathbf{i}+(3-0)\mathbf{j}+(8-0)\mathbf{k}}{\sqrt{(0-15)^2+(3-0)^2+(8-0)^2}}$$
$$= -0.8689\mathbf{i}+0.1738\mathbf{j}+0.4634\mathbf{k}$$

$$\mathbf{F} = F\mathbf{u}_{AB} = 55(-0.8689\mathbf{i}+0.1738\mathbf{j}+0.4634\mathbf{k})\ \text{lb}$$
$$= \{-47.791\mathbf{i}+9.558\mathbf{j}+25.489\mathbf{k}\}\ \text{lb}$$

Unit Vector : The unit vector along negative *x* axis and *AC* are

$$\mathbf{u}_x = -1\mathbf{i}$$

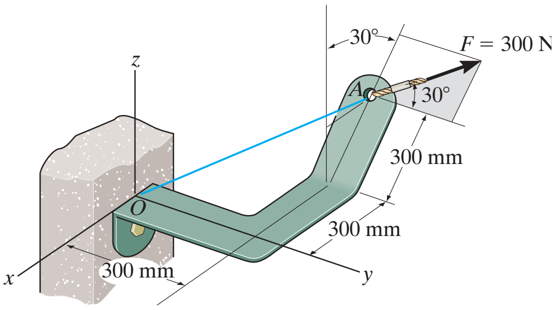
$$\mathbf{u}_{AC} = \frac{(0-15)\mathbf{i}+(-8-0)\mathbf{j}+(12-0)\mathbf{k}}{\sqrt{(0-15)^2+(-8-0)^2+(12-0)^2}}$$
$$= -0.7209\mathbf{i}-0.3845\mathbf{j}+0.5767\mathbf{k}$$

Projected Component of F :

$$F_x = \mathbf{F}\cdot\mathbf{u}_x = (-47.791\mathbf{i}+9.558\mathbf{j}+25.489\mathbf{k})\cdot(-1\mathbf{i})$$
$$= (-47.791)(-1)+9.558(0)+25.489(0)$$
$$= 47.8\ \text{lb} \qquad \textbf{Ans}$$

$$F_{AC} = \mathbf{F}\cdot\mathbf{u}_{AC} = (-47.791\mathbf{i}+9.558\mathbf{j}+25.489\mathbf{k})\cdot(-0.7209\mathbf{i}-0.3845\mathbf{j}+0.5767\mathbf{k})$$
$$= (-47.791)(-0.7209)+(9.558)(-0.3845)+(25.489)(0.5767)$$
$$= 45.5\ \text{lb} \qquad \textbf{Ans}$$

2–131. Determine the magnitudes of the projected components of the force *F* = 300 N acting along the *x* and *y* axes.



Force Vector: The force vector **F** must be determined first. From Fig. *a*,

$$\mathbf{F} = -300\sin 30^\circ \sin 30^\circ\mathbf{i} + 300\cos 30^\circ\mathbf{j} + 300\sin 30^\circ\cos 30^\circ\mathbf{k}$$
$$= [-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}]\text{N}$$

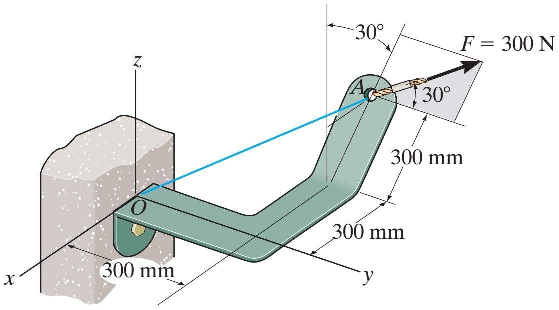
Vector Dot Product: The magnitudes of the projected component of **F** along the *x* and *y* axes are

$$F_x = \mathbf{F}\cdot\mathbf{i} = (-75\mathbf{i}+259.81\mathbf{j}+129.90\mathbf{k})\cdot\mathbf{i}$$
$$= -75(1)+259.81(0)+129.90(0)$$
$$= -75\ \text{N}$$
$$F_y = \mathbf{F}\cdot\mathbf{j} = (-75\mathbf{i}+259.81\mathbf{j}+129.90\mathbf{k})\cdot\mathbf{j}$$
$$= -75(0)+259.81(1)+129.90(0)$$
$$= 260\ \text{N}$$

The negative sign indicates that $\mathbf{F}_x$ is directed towards the negative *x* axis. Thus

$F_x = 75\text{N}, \quad F_y = 260\text{N}$ **Ans.**

***2–132.** Determine the magnitude of the projected component of the force $F = 300$ N acting along line OA.



Force and unit Vector: The force vector **F** and unit vector $\mathbf{u}_{OA}$ must be determined first. From Fig. (a)

$$\mathbf{F} = -300\sin 30^\circ \sin 30^\circ\mathbf{i} + 300\cos 30^\circ\,\mathbf{j} + 300\sin 30^\circ\cos 30^\circ\,\mathbf{k})$$

$$= \{-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}\}\text{N}$$

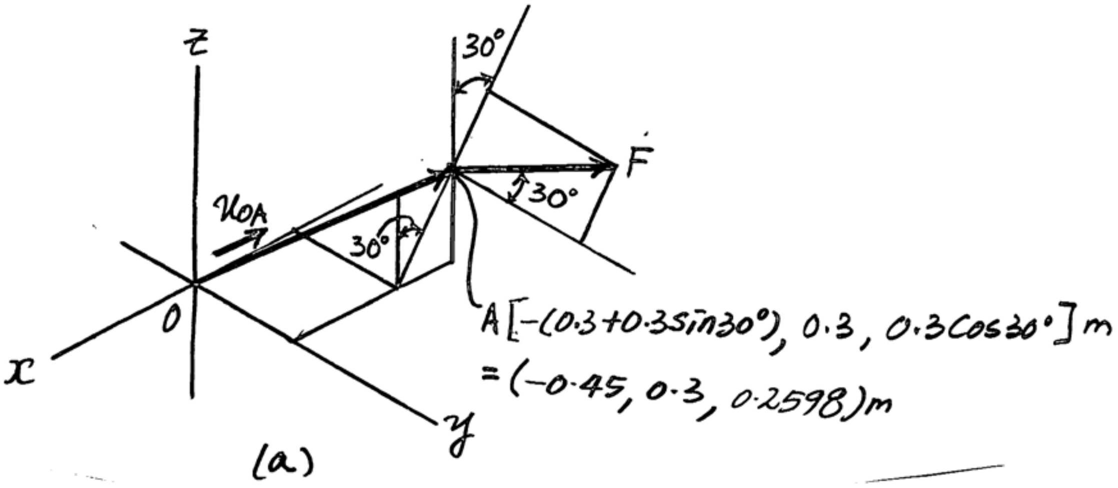
$$\mathbf{u}_{OA} = \frac{\mathbf{r}_{OA}}{r_{OA}} = \frac{(-0.45-0)\mathbf{i} + (0.3-0)\,\mathbf{j} + (0.2598-0)\mathbf{k}}{\sqrt{(-0.45-0)^2 + (0.3-0)^2 + (0.2598-0)^2}} = -0.75\mathbf{i} + 0.5\mathbf{j} + 0.4330\mathbf{k}$$

Vector Dot Product: The magnitude of the projected component of **F** along line OA is

$$F_{OA} = \mathbf{F}\cdot\mathbf{u}_{OA} = (-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k})\cdot(-0.75\mathbf{i} + 0.5\mathbf{j} + 0.4330\mathbf{k})$$

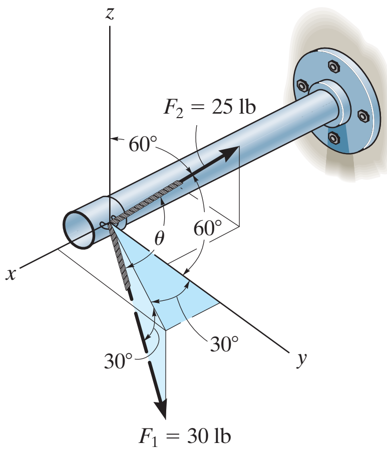
$$= (-75)(-0.75) + 259.81(0.5) + 129.90(0.4330)$$

$$= 242\text{ N} \qquad \textbf{Ans.}$$



(a)

•2–133. Two cables exert forces on the pipe. Determine the magnitude of the projected component of $\mathbf{F}_1$ along the line of action of $\mathbf{F}_2$.



Force Vector :

$$\mathbf{u}_{F_1} = \cos 30°\sin 30°\mathbf{i} + \cos 30°\cos 30°\mathbf{j} - \sin 30°\mathbf{k}$$
$$= 0.4330\mathbf{i} + 0.75\mathbf{j} - 0.5\mathbf{k}$$

$$\mathbf{F}_1 = F_1\mathbf{u}_{F_1} = 30(0.4330\mathbf{i} + 0.75\mathbf{j} - 0.5\mathbf{k})\text{ lb}$$
$$= \{12.990\mathbf{i} + 22.5\mathbf{j} - 15.0\mathbf{k}\}\text{ lb}$$

Unit Vector : One can obtain the angle $\alpha = 135°$ for $\mathbf{F}_2$ using Eq. 2–8. $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$, with $\beta = 60°$ and $\gamma = 60°$. The unit vector along the line of action of $\mathbf{F}_2$ is

$$\mathbf{u}_{F_2} = \cos 135°\mathbf{i} + \cos 60°\mathbf{j} + \cos 60°\mathbf{k} = -0.7071\mathbf{i} + 0.5\mathbf{j} + 0.5\mathbf{k}$$

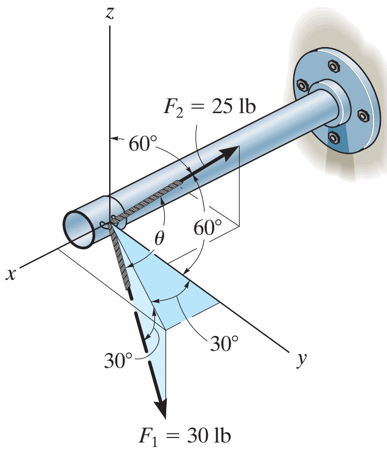
Projected Component of $\mathbf{F}_1$ Along the Line of Action of $\mathbf{F}_2$:

$$(F_1)_{F_2} = \mathbf{F}_1 \cdot \mathbf{u}_{F_2} = (12.990\mathbf{i} + 22.5\mathbf{j} - 15.0\mathbf{k}) \cdot (-0.7071\mathbf{i} + 0.5\mathbf{j} + 0.5\mathbf{k})$$
$$= (12.990)(-0.7071) + (22.5)(0.5) + (-15.0)(0.5)$$
$$= -5.44\text{ lb}$$

Negative sign indicates that the projected component $(\mathbf{F}_1)_{F_2}$ acts in the opposite sense of direction to that of $\mathbf{u}_{F_2}$.

The magnitude is $(\mathbf{F}_1)_{F_2} = 5.44$ lb. **Ans**

2–134. Determine the angle θ between the two cables attached to the pipe.



Unit Vector :

$$\mathbf{u}_{F_1} = \cos 30°\sin 30°\mathbf{i} + \cos 30°\cos 30°\mathbf{j} - \sin 30°\mathbf{k}$$
$$= 0.4330\mathbf{i} + 0.75\mathbf{j} - 0.5\mathbf{k}$$

$$\mathbf{u}_{F_2} = \cos 135°\mathbf{i} + \cos 60°\mathbf{j} + \cos 60°\mathbf{k}$$
$$= -0.7071\mathbf{i} + 0.5\mathbf{j} + 0.5\mathbf{k}$$

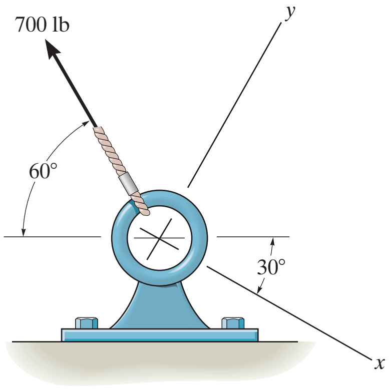
The Angles Between Two Vectors θ:

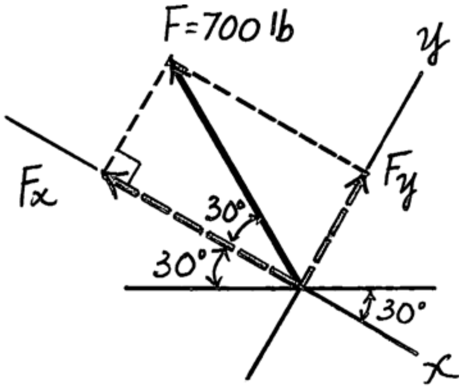
$$\mathbf{u}_{F_1} \cdot \mathbf{u}_{F_2} = (0.4330\mathbf{i} + 0.75\mathbf{j} - 0.5\mathbf{k}) \cdot (-0.7071\mathbf{i} + 0.5\mathbf{j} + 0.5\mathbf{k})$$
$$= 0.4330(-0.7071) + 0.75(0.5) + (-0.5)(0.5)$$
$$= -0.1812$$

Then,

$$\theta = \cos^{-1}\left(\mathbf{u}_{F_1} \cdot \mathbf{u}_{F_2}\right) = \cos^{-1}(-0.1812) = 100° \qquad \textbf{Ans}$$

2–135. Determine the *x* and *y* components of the 700-lb force.

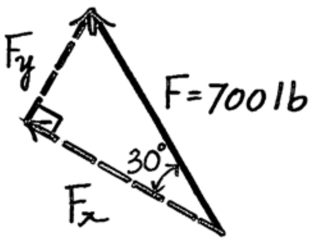




(a)

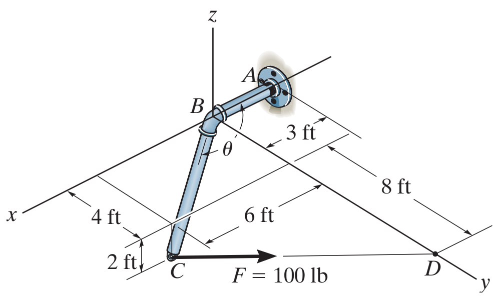
$$F_x = -700 \cos 30° = -606 \text{ lb} \quad \textbf{Ans}$$

$$F_y = 700 \sin 30° = 350 \text{ lb} \quad \textbf{Ans}$$



(b)

***2–136.** Determine the magnitude of the projected component of the 100-lb force acting along the axis *BC* of the pipe.



Force Vector :

$$\mathbf{u}_{CD} = \frac{(0-6)\mathbf{i} + (12-4)\mathbf{j} + [0-(-2)]\mathbf{k}}{\sqrt{(0-6)^2 + (12-4)^2 + [0-(-2)]^2}}$$
$$= -0.5883\mathbf{i} + 0.7845\mathbf{j} + 0.1961\mathbf{k}$$

$$\mathbf{F} = F\mathbf{u}_{CD} = 100(-0.5883\mathbf{i} + 0.7845\mathbf{j} + 0.1961\mathbf{k})$$
$$= \{-58.835\mathbf{i} + 78.446\mathbf{j} + 19.612\mathbf{k}\} \text{ lb}$$

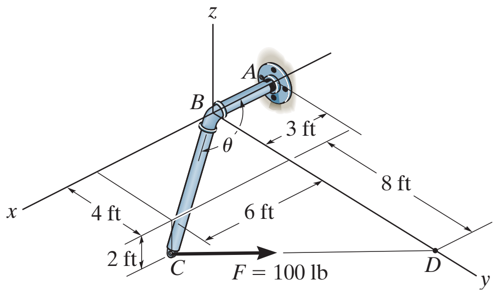
Unit Vector : The unit vector along *CB* is

$$\mathbf{u}_{CB} = \frac{(0-6)\mathbf{i} + (0-4)\mathbf{j} + [0-(-2)]\mathbf{k}}{\sqrt{(0-6)^2 + (0-4)^2 + [0-(-2)]^2}}$$
$$= -0.8018\mathbf{i} - 0.5345\mathbf{j} + 0.2673\mathbf{k}$$

Projected Component of F Along CB :

$$F_{CB} = \mathbf{F} \cdot \mathbf{u}_{CB} = (-58.835\mathbf{i} + 78.446\mathbf{j} + 19.612\mathbf{k}) \cdot (-0.8018\mathbf{i} - 0.5345\mathbf{j} + 0.2673\mathbf{k})$$
$$= (-58.835)(-0.8018) + (78.446)(-0.5345) + (19.612)(0.2673)$$
$$= 10.5 \text{ lb} \qquad \textbf{Ans}$$

•2–137. Determine the angle θ between pipe segments *BA* and *BC*.



Position Vector :

$$\mathbf{r}_{BA} = \{-3\mathbf{i}\} \text{ ft}$$

$$\mathbf{r}_{BC} = \{(6-0)\mathbf{i} + (4-0)\mathbf{j} + (-2-0)\mathbf{k}\} \text{ ft}$$
$$= \{6\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}\} \text{ ft}$$

The magnitudes of the position vectors are

$$r_{BA} = 3.00 \text{ ft} \qquad r_{BC} = \sqrt{6^2 + 4^2 + (-2)^2} = 7.483 \text{ ft}$$

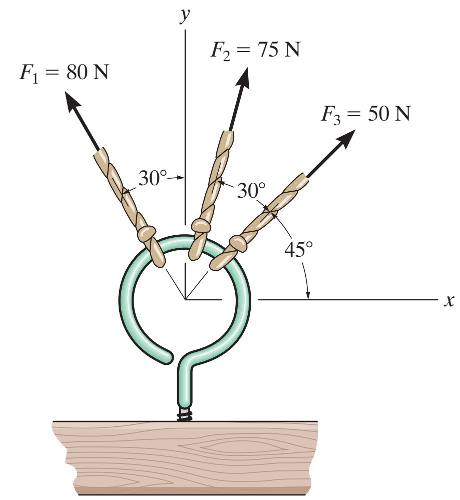
The Angle Between Two Vectors θ:

$$\mathbf{r}_{BA} \cdot \mathbf{r}_{BC} = (-3\mathbf{i}) \cdot (6\mathbf{i} + 4\mathbf{j} - 2\mathbf{k})$$
$$= (-3)(6) + (0)(4) + 0(-2)$$
$$= -18.0 \text{ ft}^2$$

Then,

$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_{BA} \cdot \mathbf{r}_{BC}}{r_{BA} r_{BC}}\right) = \cos^{-1}\left[\frac{-18.0}{3.00(7.483)}\right] = 143^\circ \qquad \textbf{Ans}$$

2–138. Determine the magnitude and direction of the resultant $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$ of the three forces by first finding the resultant $\mathbf{F}' = \mathbf{F}_1 + \mathbf{F}_3$ and then forming $\mathbf{F}_R = \mathbf{F}' + \mathbf{F}_2$. Specify its direction measured counterclockwise from the positive x axis.



$$F' = \sqrt{(80)^2 + (50)^2 - 2(80)(50)\cos 105^\circ} = 104.7\ \text{N}$$

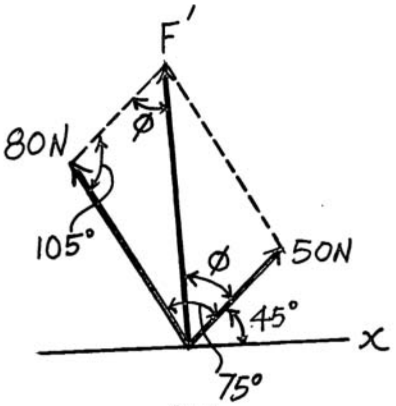
$$\frac{\sin\phi}{80} = \frac{\sin 105^\circ}{104.7}; \quad \phi = 47.54^\circ$$

$$F_R = \sqrt{(104.7)^2 + (75)^2 - 2(104.7)(75)\cos 162.46^\circ}$$

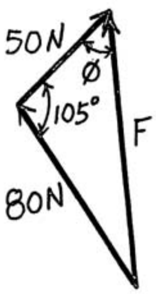
$F_R = 177.7 = 178$ N **Ans**

$$\frac{\sin\beta}{104.7} = \frac{\sin 162.46^\circ}{177.7}; \quad \beta = 10.23^\circ$$

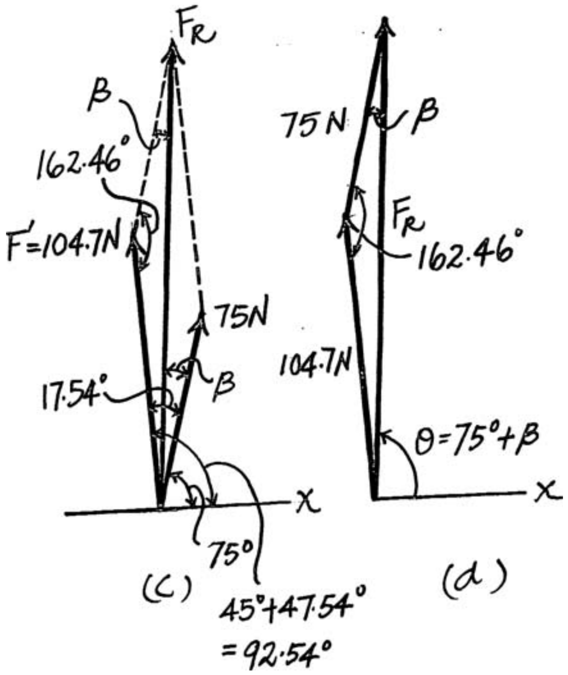
$\theta = 75^\circ + 10.23^\circ = 85.2^\circ$ **Ans**



(a)



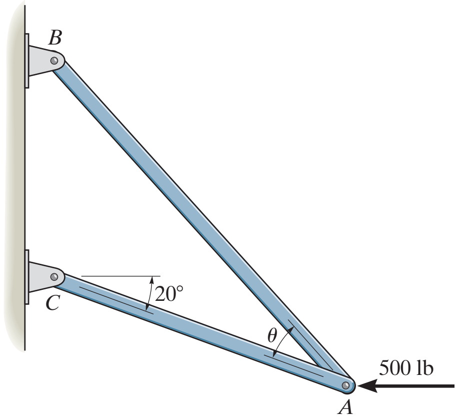
(b)



(c)

(d)

2–139. Determine the design angle θ ($\theta < 90°$) between the two struts so that the 500-lb horizontal force has a component of 600 lb directed from A toward C. What is the component of force acting along member BA?



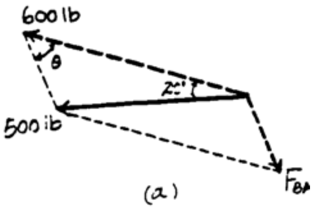
The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*.

Applying the law of cosines to Fig. *b*,

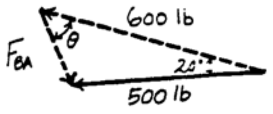
$$F_R = \sqrt{500^2 + 600^2 - 2(500)(600)\cos 20°}$$
$$= 214.91 \text{ lb} = 215 \text{ lb} \qquad \textbf{Ans.}$$

Applying the law of sines to Fig. *b* and using this result yields

$$\frac{\sin\theta}{500} = \frac{\sin 20°}{214.91} \qquad \theta = 52.7° \qquad \textbf{Ans.}$$

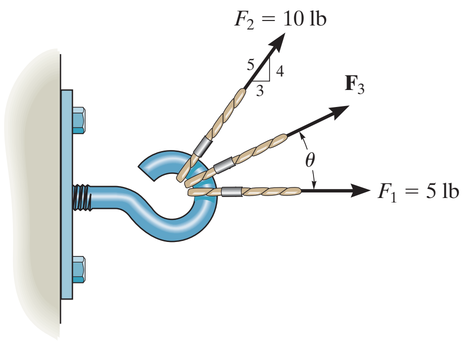


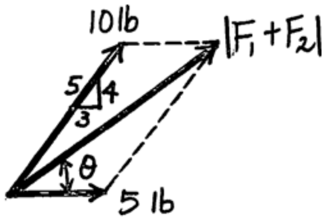
(a)



(b)

***2–140.** Determine the magnitude and direction of the *smallest* force $\mathbf{F}_3$ so that the resultant force of all three forces has a magnitude of 20 lb.





(a)

F_3 is minimum:

$F_3 = 20 - |F_1 + F_2|$

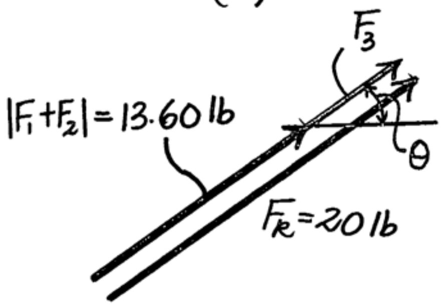
$F_1 + F_2 = \left(5 + 10\left(\frac{3}{5}\right)\right)\mathbf{i} + \left(10\left(\frac{4}{5}\right)\right)\mathbf{j} = 11\mathbf{i} + 8\mathbf{j}$

$|F_1 + F_2| = \sqrt{11^2 + 8^2} = 13.601$ lb

$\theta = \tan^{-1}\left(\frac{8}{11}\right) = 36.0^\circ$ Ans

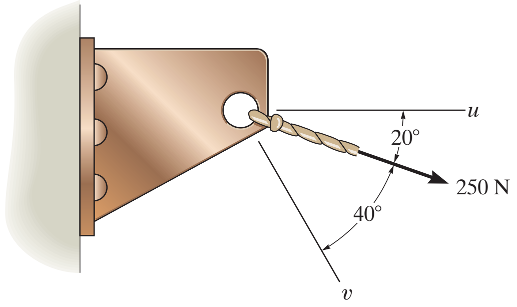
Thus

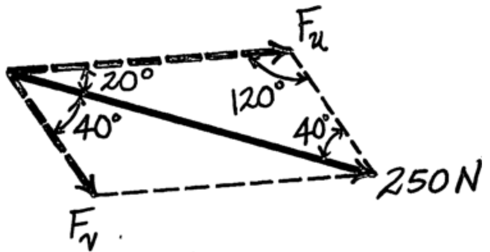
$(F_3)_{min} = 20 - 13.601 = 6.40$ lb Ans



(b)

•2–141. Resolve the 250-N force into components acting along the *u* and *v* axes and determine the magnitudes of these components.

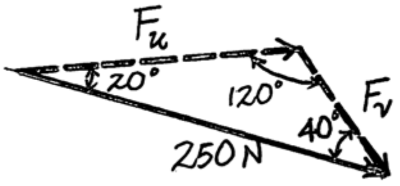




(a)

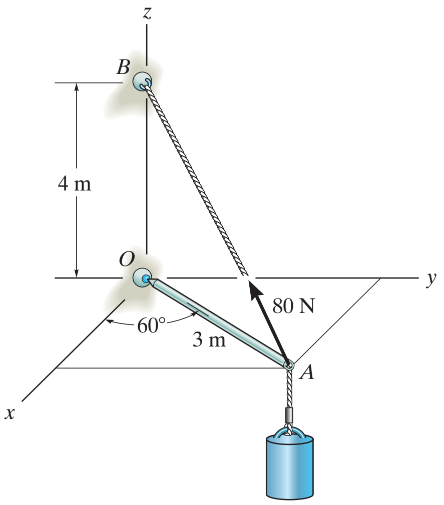
$$\frac{250}{\sin 120^\circ} = \frac{F_u}{\sin 40^\circ}; \qquad F_u = 186 \text{ N} \qquad \text{Ans}$$

$$\frac{250}{\sin 120^\circ} = \frac{F_v}{\sin 20^\circ}; \qquad F_v = 98.7 \text{ N} \qquad \text{Ans}$$



(b)

2–142. Cable *AB* exerts a force of 80 N on the end of the 3-m-long boom *OA*. Determine the magnitude of the projection of this force along the boom.



Vector Analysis :

$$\mathbf{F} = 80\left(\frac{\mathbf{r}_{AB}}{r_{AB}}\right) = 80\left(-\frac{3\cos 60^\circ}{5}\mathbf{i} - \frac{3\sin 60^\circ}{5}\mathbf{j} + \frac{4}{5}\mathbf{k}\right)$$

$$= -24\mathbf{i} - 41.57\mathbf{j} + 64\mathbf{k}$$

$$\mathbf{u}_{AO} = -\cos 60^\circ\,\mathbf{i} - \sin 60^\circ\,\mathbf{j} = -0.5\mathbf{i} - 0.866\mathbf{j}$$

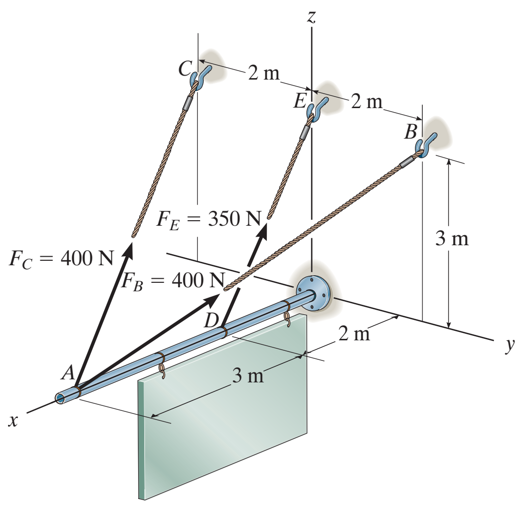
$$\text{Proj}\,F = \mathbf{F}\cdot\mathbf{u}_{AO} = (-24)(-0.5) + (-41.57)(-0.866) + (64)0 = 48.0 \text{ N} \qquad \text{Ans}$$

Scalar Anaysis :

$$\text{Angle } OAB = \tan^{-1}\left(\frac{4}{3}\right) = 53.13^\circ$$

$$\text{Proj}\,F = 80\cos 53.13^\circ = 48.0 \text{ N} \qquad \text{Ans}$$

2–143. The three supporting cables exert the forces shown on the sign. Represent each force as a Cartesian vector.



Unit Vector:

$$\mathbf{r}_{AB} = \{(0-5)\mathbf{i} + (2-0)\mathbf{j} + (3-0)\mathbf{k}\}\ \text{m} = \{-5\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}\}\ \text{m}$$

$$r_{AB} = \sqrt{(-5)^2 + 2^2 + 3^2} = 6.164\ \text{m}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{-5\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}}{6.164} = -0.8111\mathbf{i} + 0.3244\mathbf{j} + 0.4867\mathbf{k}$$

$$\mathbf{r}_{AC} = \{(0-5)\mathbf{i} + (-2-0)\mathbf{j} + (3-0)\mathbf{k}\}\ \text{m} = \{-5\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}\}\ \text{m}$$

$$r_{AC} = \sqrt{(-5)^2 + (-2)^2 + 3^2} = 6.164\ \text{m}$$

$$\mathbf{u}_{AC} = \frac{\mathbf{r}_{AC}}{r_{AC}} = \frac{-5\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}}{6.164} = -0.8111\mathbf{i} - 0.3244\mathbf{j} + 0.4867\mathbf{k}$$

$$\mathbf{r}_{DE} = \{(0-2)\mathbf{i} + (0-0)\mathbf{j} + (3-0)\mathbf{k}\}\ \text{m} = \{-2\mathbf{i} + 3\mathbf{k}\}\ \text{m}$$

$$r_{DE} = \sqrt{(-2)^2 + 3^2} = 3.605\ \text{m}$$

$$\mathbf{u}_{DE} = \frac{\mathbf{r}_{DE}}{r_{DE}} = \frac{-2\mathbf{i} + 3\mathbf{k}}{3.605} = -0.5547\mathbf{i} + 0.8321\mathbf{k}$$

Force Vector:

$$\mathbf{F}_B = F_B\mathbf{u}_{AB} = 400\{-0.8111\mathbf{i} + 0.3244\mathbf{j} + 0.4867\mathbf{k}\}\ \text{N}$$
$$= \{-324.44\mathbf{i} + 129.78\mathbf{j} + 194.67\mathbf{k}\}\ \text{N}$$
$$= \{-324\mathbf{i} + 130\mathbf{j} + 195\mathbf{k}\}\ \text{N} \qquad \textbf{Ans}$$

$$\mathbf{F}_C = F_C\mathbf{u}_{AB} = 400\{-0.8111\mathbf{i} - 0.3244\mathbf{j} + 0.4867\mathbf{k}\}\ \text{N}$$
$$= \{-324.44\mathbf{i} - 129.78\mathbf{j} + 194.67\mathbf{k}\}\ \text{N}$$
$$= \{-324\mathbf{i} - 130\mathbf{j} + 195\mathbf{k}\}\ \text{N} \qquad \textbf{Ans}$$

$$\mathbf{F}_E = F_E\mathbf{u}_{DE} = 350\{-0.5547\mathbf{i} + 0.8321\mathbf{k}\}\ \text{N}$$
$$= \{-194.15\mathbf{i} + 291.22\mathbf{k}\}\ \text{N}$$
$$= \{-194\mathbf{i} + 291\mathbf{k}\}\ \text{N} \qquad \textbf{Ans}$$